AN INVESTIGATION OF RECIPROCITY IN THE EXPONENTIAL ASSEMBLY

JOHN F. PEARSON
AND
ROBERT B. SIMS

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JOHN F. PEARSON, JR.

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ROBERT B. SIMS

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Signature of Authors ______________________

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Department of Naval Architecture and Marine Engineering
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Certified by ______________________

Thesis Supervisor

Accepted by ______________________

Chairman, Departmental Committee on Graduate Students

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ABSTRACT

The purpose of this thesis is to investigate a proposed reciprocal method for measurements within an exponential assembly. It is shown that the usual positions of source and detector may be reversed, provided that geometric and macroscopic nuclear correspondence of these two elements is obtained. Reciprocal and conventional measurements were made for neutron density under various conditions of loading and source-detector orientation. Certain basic pile parameters were calculated using elementary pile theory in order to correlate the two methods of measurement.

All measurements were made on the Massachusetts Institute of Technology subcritical graphite pile. Both indium foil and BF_3 counting methods were employed. Results obtained definitely confirm the experimental utility and theoretical accuracy of reciprocal measurements. Certain worthwhile advantages are associated with the proposed technique, and these are examined in detail.

Thesis Supervisor: Theos J. Thompson, Ph.D

Title: Professor of Nuclear Engineering; Director of M.I.T. Reactor

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NOMENCLATURE

b_{mn}	Relaxation length of mn th harmonic
γ_{mn}	$1/b_{mn}$; slope of $\log_e(N_{th})$ vs. z
τ	Fermi Age
L	Diffusion length
B_m^2	Material buckling
Φ_{mn}	Neutron flux density corresponding to the mn th harmonic
N_{th}	Counting rate due to thermal neutrons
x	East-west coordinate for measurement of pile dimensions
y	North-south coordinate of pile dimensions
z	Vertical coordinate
a	Extrapolated length in x direction
b	Extrapolated length in y direction
c	Extrapolated length in z direction
d	Linear extrapolation distance
N_b	Counting rate for bare foils or BF_3 tube, no external source
N_c	Counting rate, CD covered, no source
N_B	Counting rate, bare with external source
N_C	Counting rate, Cd covered, with source
CR	Cd ratio $\frac{N_B}{N_C}$ or $\frac{N_b}{N_c}$
M	Multiplication ratio $\frac{N_{th}\text{ (loaded)}}{N_{th}\text{ (unloaded)}}$
Keff	Effective pile multiplication
K_∞	Infinite multiplication factor

I. INTRODUCTION

A. Historical Summary

The exponential experiment, in its conventional form, is perhaps the classic investigation relating to elementary reactor theory. Methods of measurement employed and experimental data obtained from exponential assemblies were of vital importance during the early period of nuclear reactor development. The first exponential experiment was conceived by Enrico Fermi and his associates while conducting their well known University of Chicago studies.

Continuing use of these experiments for preliminary reactor design information and for student instruction attests to the fact that exponential methods have a permanent place in the field of reactor technology.

When intended for design data, the exponential assembly is constructed with lattice dimensions identical with those of a proposed critical reactor. Overall dimensions are considerably less, being perhaps one-third of those required for criticality. Hence an exponential system will not sustain a fission chain reaction because of the relatively great neutron leakage which exists. It is possible, however, to attain steady state flux conditions by means of an external neutron source--usually a Pu-Be or Po-Be cylinder or a reactor thermal column. The wave equation for a critical system under steady

state conditions

$$\nabla^2\Phi + B_m^2\Phi = 0 \tag{1}$$

is closely approximated in the exponential, or subcritical, assembly. The general solution to this equation, following the development of Glasstone & Edlund (G-1,ch.9) is

$$\Phi = \sum_m \sum_n A_{mn} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \sinh \gamma_{mn}(c-z) \tag{2}$$

If the lattice has a plane source of thermal neutrons, contributions of harmonic terms to the flux can be neglected. The general solution then becomes

$$\Phi = A_{11} \cos\frac{\pi x}{a} \cos\frac{\pi y}{b} \sinh \gamma (c-z) \tag{3}$$

Thermal flux distribution along any line parallel to the z-axis, then, is given by

$$\Phi(z) = F \sinh \gamma(c-z) = C e^{-\gamma z}\left[1 - e^{-2\gamma(c-z)}\right] \tag{4}$$

If z does not closely approach c, i.e. if the "end correction" term in the bracket is about unity, then

$$\Phi(z) = C e^{-\gamma z} \tag{5}$$

Hence, in the system under consideration, neutron flux in the z-direction diminishes exponentially with distance from the origin of coordinates. It is this feature which has given the name "exponential" to such subcritical assemblies.

B. Application of Reciprocity Principles

The theorem of reciprocity, in its various forms, has been valuable in several areas of nuclear physics and reactor engineering. Notable was its application by Bethe and Beyster (B-3, B-4) for measurement of inelastic neutron scattering. A number of other instances are described in the literature.

Direct investigation of neutron flux density historically has been a primary tool of the experimenter. By this means the various parameters required for reactor design are obtained. In an exponential assembly, flux measurements are made in a reproducing lattice supplied with neutrons from an external source. These methods have been employed for many years, and the concept of neutron flux in this configuration is well established.

It is not illogical, however, that the original concept equally well might have been in terms of the adjoint flux or importance function. This function is so named because the total progeny which result from a single neutron introduced at a given point in a reactor is a measure of the "importance" of this neutron in sustaining the chain reaction. Simply speaking, a neutron introduced at the center of a reactor, along with its daughter neutrons, is less likely to leak from the multiplying region than is the case for a corresponding neutron introduced near an edge of the reactor. In the case of a thermal, one-group reactor--and only in this case--the importance concept predicts an adjoint flux distribution which corresponds exactly to the flux distribution of

thermal neutrons. Consequently, in a static, one-group assembly, neutron flux and adjoint flux become identical functions of position.

If consideration is restricted to the one-group reactor, the adjoint concept becomes equally acceptable with the conventional idea of neutron flux, after a brief period of orientation. Apart from its wider application in the field of multi-group theory, preference is accorded to the classical concept because of general familiarity and a well established mode of thought. However, if the experimenter is thinking in terms of the importance function, certain modifications to standard experimental procedures become plausible.

This thesis is concerned with one of these possible modifications which has been proposed by Lewins (L-3). In accordance with his suggested application of reciprocity, the classical exponential experiment is reversed. A neutron source is placed within the loaded lattice, as would be done intuitively if the experimenter were thinking in terms of adjoint flux. A BF_3 neutron detector is positioned within the graphite pedestal where neutron density becomes a direct measure of importance function at the reciprocal source location. In a broad sense it is the purpose of this thesis to investigate all facets of such a reciprocal arrangement, with emphasis upon accuracy, precision, and experimental utility. Theory indicates that reciprocal methods will give results equally valid with those obtained by conventional means. It

suggests that certain advantages will be associated with reciprocal techniques.

C. The Conventional Exponential Experiment

In the usual subcritical experiment, detectors are placed within a uniformly loaded lattice in positions sufficient to define the vertical and horizontal distribution of neutron density. These detectors typically consist of indium foils or boron fluoride gas-filled counters. Since the thermal flux distribution is often of primary interest, additional measurements may be taken under cadmium covers to separate the thermal and epithermal components. It is generally acknowledged that foil measurements provide greater experimental accuracy than do BF_3 data. This is due largely to the fact that foils cause a much smaller local flux perturbation. Under certain circumstances, however, experimental results obtained from BF_3 measurements may be equally acceptable.

Primary neutrons are supplied alternatively by a reactor thermal column or self-contained sources producing neutrons by an (α-n) or (γ-n) reaction. The former method possesses a distinct advantage in that the neutrons, as supplied, are completely thermalized and are essentially in the form of a plane source. Use of self-contained sources is generally simpler, but harmonic and slowing down effects must be carefully considered.

Within limits imposed by pile characteristics and equipment available, a number of experimental measurements may be

accomplished on the exponential pile. A partial list includes: material buckling (C-2, W-3), diffusion length (R-1, R-2), Fermi Age (H-3), effective size (C-3), thermal utilization (R-2), effective multiplication (C-2), neutron temperature (H-4), albedo (G-1), and reflector savings. These quantities are essential in determination of criticality conditions for a reactor but are extremely difficult to determine with sufficient accuracy from theoretical calculations alone. Furthermore, the basic exponential experiments enable a student to acquire a feeling for reactor physics which is vital to the nuclear engineer.

D. The M.I.T. Exponential Pile

Photographs of the M.I.T. exponential assembly are shown in figures 1 and 2. A detailed drawing appears in figure 3. Overall dimensions are as follows:

x-dimension (east-west)	90.75" (230.5 cm)	
y-dimension (north-south)	91.00" (231.1 cm)	
z-dimension (vertical)	116.00" (294.7 cm)	overall
	25.25" (64.2 cm)	pedestal
	90.75" (230.5 cm)	lattice

For uranium loading, 144 fuel channels are provided in the lattice region, each channel 1.25" in diameter and extending full width in the y-direction. Natural uranium fuel slugs are standard AEC issue for educational use--1.087"OD and 8.375" long. Nine slugs are loaded in each channel, providing an essentially uniform loading on a 7.25" (18.415 cm) lattice pitch. The assembly is fitted with sliding graphite stringers on centerline and extending in both the x- and y-directions.

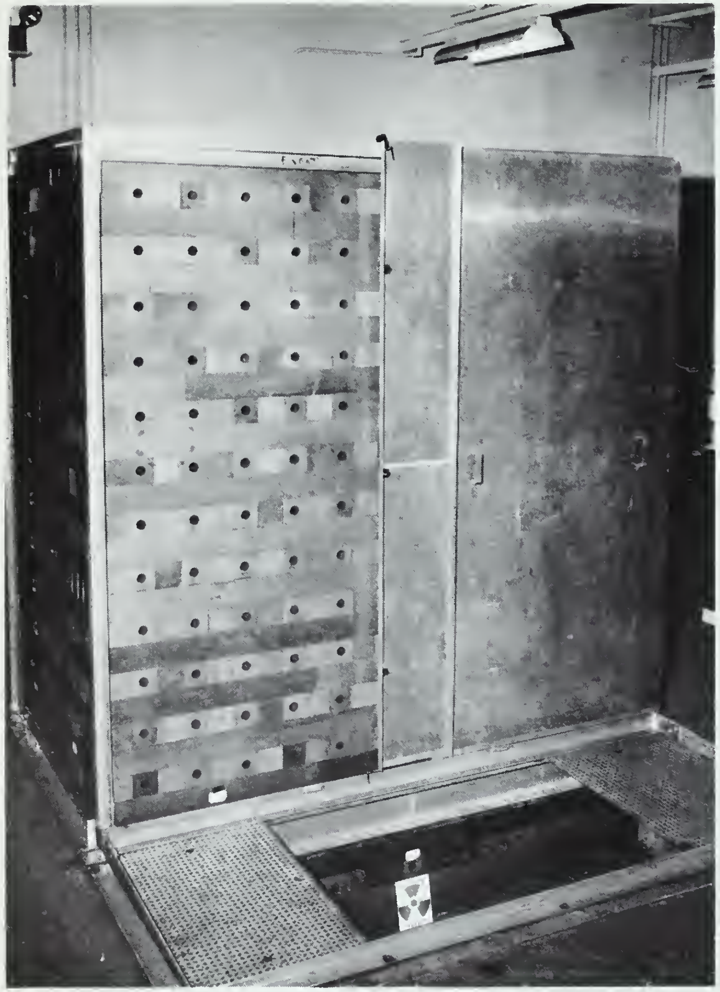


FIGURE I. NORTH FACE OF EXPONENTIAL ASSEMBLY. SOURCE BLOCKS CAN BE SEEN IN PIT, LOWER RIGHT.

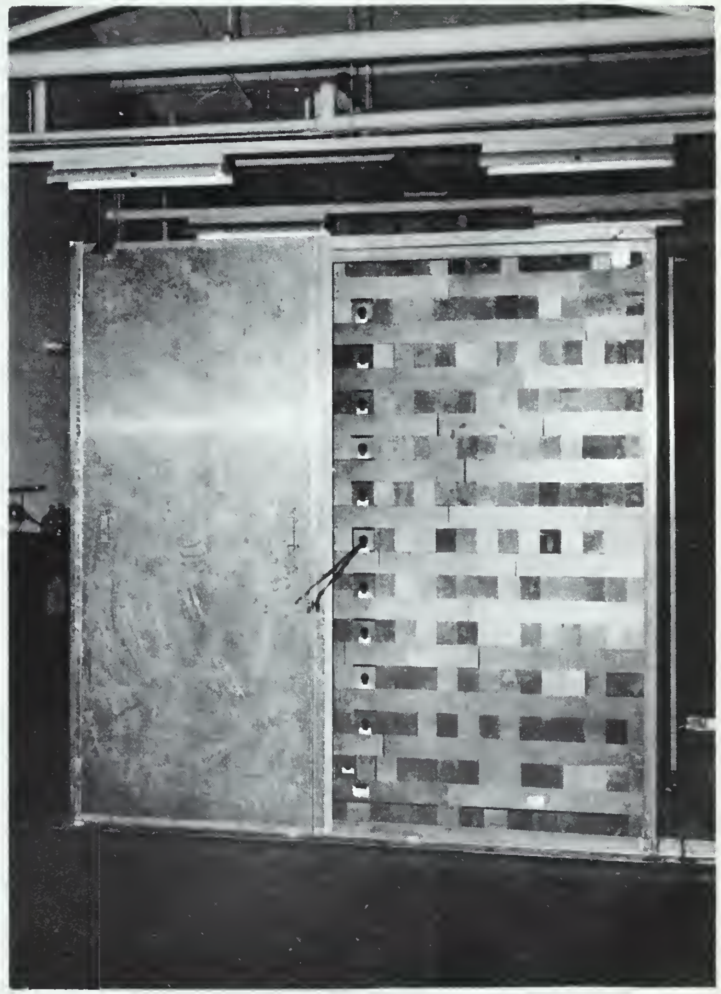


FIGURE 2. EAST FACE OF PILE, SHOWING SOURCE POSITIONING ROD IN DETECTOR CHANNEL.

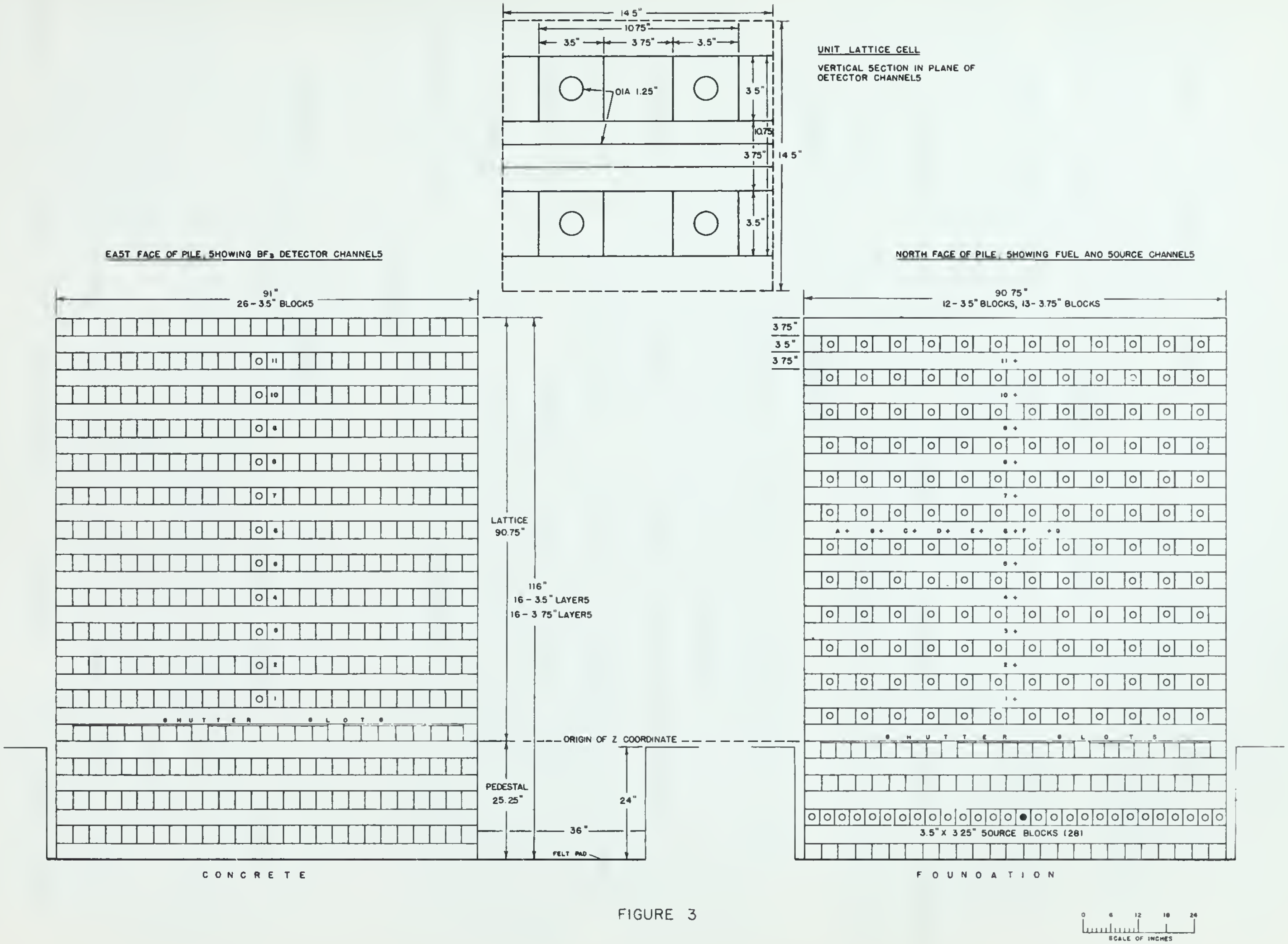


FIGURE 3

THE M.I.T. EXPONENTIAL ASSEMBLY - BLDG. NW12

Measurements within the lattice may be made with foils or BF_3 counters. The details of pile dimensions and the coordinates of measuring positions are listed in Table I of Appendix I. A summary of all pertinent physical characteristics of the pile is included in Table II.

There are slots at the lattice-pedestal interface for insertion of cadmium shutters under those conditions when it is desired to minimize passage of thermal neutrons between these regions. Shutters for this purpose were fabricated as a by-product of this thesis.

The M.I.T. exponential assembly is physically far removed from the reactor and is not designed for utilization of a thermal column source. A single level of source channels is provided in the pedestal, giving two degrees of freedom in source position. A diamond source array, which will minimize harmonic effects, is possible in the conventional pile configuration, though such an arrangement was not used for the present experiment.

E. Purposes of the Thesis

It was the primary purpose of this thesis to test the validity of the theorem of reciprocity as applied to exponential measurements. To do this it was first necessary to devise experimental methods which would permit these measurements to be made in the reciprocal sense. Following this, certain classic pile parameters were to be measured in the M.I.T. assembly by both conventional and reciprocal methods,

permitting a direct comparison. This comparison was to include such factors as statistical accuracy, validity and precision of results, and ease of experimentation.

Particular attention was given to the physical configuration of source and detector in order that the nuclear characteristics of these components might correspond as nearly as possible. Having obtained this correspondence, one could logically expect similar perturbations of the flux distribution by both source and detector.

The counter assembly was examined for anisotropy in order to validate measurements taken near the pile boundary during horizontal traverses. To accomplish this, a directional stream of fast neutrons was obtained and the counter placed in this stream. By taking a series of measurements at various angles between beam direction and counter axis, it was determined that the degree of anisotropy inherent in the BF_3 tube was not sufficient to affect measurements within the pile.

Since the reciprocal technique involves much more handling of the neutron source than is necessary for conventional experiments, it was important to devise a safe yet practical means for positioning the source. In this connection, radiation safety comprised an integral phase of the investigation.

There are certain results to be anticipated from reciprocal measurements beyond those relating to experimental accuracy and ease of experimentation. In addition to verification of the one-group reciprocity within a subcritical pile,

a rational, easily visualized substance might be given to the concept of adjoint flux.

With such purposes in mind the following investigation was made.

II. PROCEDURE

A. Indium Foil Measurements

The intended purpose of this phase of experimentation was to establish "control" values for pile parameters such as buckling and diffusion length, against which BF_3 results might be compared. Foil data were notably unsuccessful in serving this purpose, due largely to low activations and a consequent lack of statistical reliability. A complete discussion of equipment, procedure, and results is included in Appendix C.

B. Problems Unique to the Reciprocal Method

Two basic difficulties arise in reciprocal measurement which are not in common with the usual type of exponential experiment. The first of these is associated with the effects of source and BF_3 detector upon pile flux; the second is concerned with a possible radiation safety hazard incurred by frequent handling of the neutron source.

In developing experimental procedures for this investigation, several means for attaining source and detector correspondence were considered. Generally speaking, these fell into one of two categories. On the one hand, differences in neutron absorption might be minimized by fitting both source and detector within suitable containers of large macroscopic absorption. This method must accept very large perturbations of the flux and consequently must sacrifice a considerable degree of accuracy in defining its distribution. A second

approach to this problem consists of matching certain unavoidable differences between source and detector as closely as possible, accepting whatever variance remains. This latter method was ultimately employed.

Two Pu-Be sources were used, with a combined overall length closely approximating that of the active portion of the BF_3 tube. A source container was fabricated in the shop from aluminum tubing. Since the container is thin and the neutron cross section for aluminum small, it is plausible to neglect any effect of this component upon the neutron flux distribution. To this container was attached a base assembly, complete with coaxial cable, from an unusable BF_3 tube of the same type as that employed for counting. For epithermal measurements, suitable cadmium jackets of identical weight were made for both source and detector. Figure 4 is a photograph of these assemblies as they finally evolved.

In this manner good correspondence was obtained, assuming the total macroscopic cross section of source cylinders and gas-filled portion of the tube to be identical. This assumption, though by no means precise, is not a bad one. Calculations were made to determine the total macroscopic cross sections of the active portion of the BF_3 tube and of the neutron sources. The computations showed that the primary neutron absorbers, namely B^{10} and Pu^{239}, had total neutron absorption cross sections of the order of .6 cm^2 and 8 cm^2 respectively. It is felt that the desirably small flux perturbation which arises is adequate to justify a slight lack of

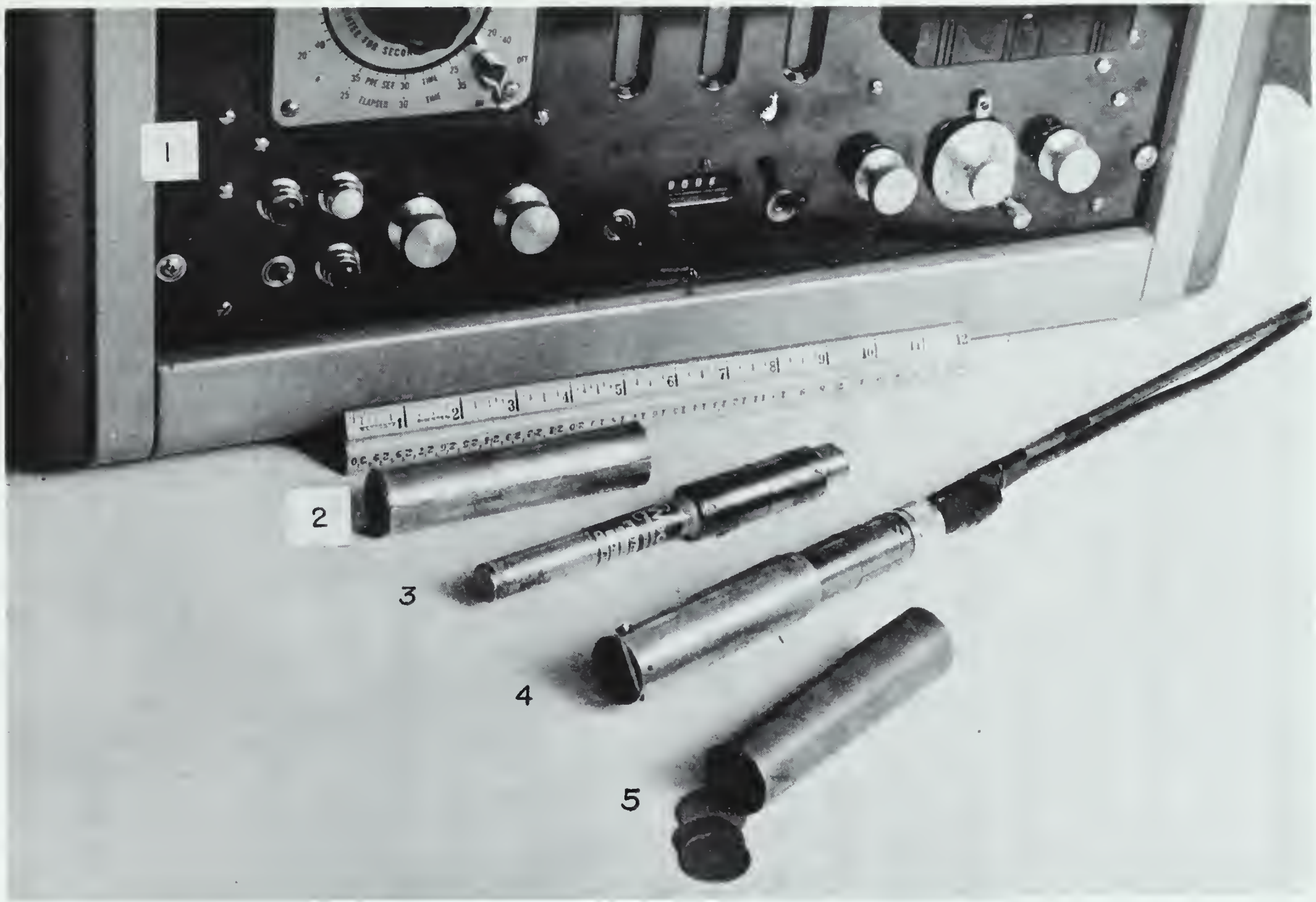


FIGURE 4. EQUIPMENT USED IN EXPERIMENT:
1. BINARY COUNTER-SCALER
2. CADMIUM SHIELD FOR BF_3 TUBE
3. BF_3 DETECTOR
4. SOURCE CARRIER, ATTACHED TO POSITIONING ROD
5. CADMIUM SLEEVE FOR SOURCE CARRIER

correspondence between source and detector.

The second basic difficulty concerns personnel safety during source handling. Since the source must be moved once for each separate positioning of the detector in a conventional experiment, it was necessary to devise an effective yet simple method of precise control which would involve a minimum of exposure to the source. To accomplish this, a section of 3/8" aluminum rod approximately six feet long was connected to the source container. A corresponding length was attached to the detector to insure similar effects upon flux distribution. These handling rods afforded a means of positioning source and detector from the pile surface, providing excellent accuracy of position. It is estimated that all measurement locations are correct to 1/32" or less. Hence the possible effect of positioning error is not considered in evaluating the results of this investigation.

Film badges and pocket dosimeters were worn by both thesis participants throughout all experimental phases, thus providing a final evaluation of radiation safety aspects.

C. BF_3 Measurements -- Pile Loaded

This phase of experimental work consisted of eight complete sets of vertical and horizontal traverses within the reproducing lattice. Four series of measurements were made which corresponded exactly to the classic experiment for material buckling. These consisted of BF_3 traverses with the

detector alternately bare and shielded by cadmium for both source and no-source conditions. Measurements were made at each of seventeen different positions within the assembly, eleven of these defining the vertical flux and six being associated with the horizontal distribution.

Following this, another series of four measurements was made in the reciprocal configuration. Two sets consisted of bare and cadmium covered source traverses throughout the seventeen lattice positions. The detector remained in a fixed pedestal location, being alternately bare and sheathed with cadmium. Measurement of reciprocal "background" was accomplished simply by two counting periods -- bare and cadmium -- with source removed from the lattice and detector stationary within the pedestal.

A general outline of the conditions under which BF_3 counts were made in the loaded pile, with reference to appropriate data tables in Appendix I, is given here; stations referred to are shown clearly in Figure 3, and their coordinates are given in Table I, Appendix I.

Run 1 Source in pedestal: count with detector at each of stations 1 - 11 and a - g with and without cadmium covers in place. (34 counts)

Run 2 No source: same procedure as above, to determine thermal neutron background from spontaneous fission. (34 counts)

Run 3 Detector in pedestal: count with source at each of stations 1 - 11 and a - g, with and without cadmium covers in place. (34 counts)

Run 4 Detector in pedestal: count with no source in lattice, with and without cadmium covers in place, to determine the spontaneous fission thermal neutron background. (2 counts)

(Data from runs 1 - 4 appear in Table V, Appendix I).

As a general policy, duration of counting was adjusted for each position in order that final statistical error should not exceed 1%. Data were corrected for resolving time of the BF_3 tube whenever these corrections approached statistical significance (E-1).

Conventional and reciprocal data were adjusted for end leakage and diffusion harmonics by iterative methods. After considerable thought it was determined that these corrections are identical for corresponding conventional or reciprocal measurements. This is equivalent to stating that the corrections are functions of position and relaxation length alone, applying equally to the one-group distributions of flux and adjoint flux. The correction for diffusion harmonics was applied to a two-region system, employing different harmonic attenuation within pedestal and lattice portions of the pile. The basic procedure employed is detailed by Glasstone and Edlund (G-1, p.126). A modification was introduced by separate consideration of harmonic attenuation with pedestal and lattice portions of the pile. Since these two regions are of totally different character when the lattice is loaded with uranium, it is evident that the relaxation length for a given harmonic must change abruptly at the interface.

Assuming the pedestal to consist of solid graphite, κ^2 from the unloaded data was adjusted for density variation as later shown by equation (13) and γ_{mn} calculated from

$$\gamma_{mn}^2 = \kappa^2 + \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] \qquad (6)$$

Thence a harmonic correction factor may be obtained from the relationship

$$C_H = 1 + \left[\frac{\gamma_{11}}{\gamma_{mn}} e^{-(\gamma_{mn} - \gamma_{11})z}\right] \qquad (7)$$

and the fundamental mode obtained by division of the measured density by this factor. By substitution of the appropriate value for z in equation (7), the magnitude of C_H at the interface is obtained.

Within the reproducing region, γ_{mn} is derived from

$$\gamma_{mn}^2 = \left[\left(\frac{m\pi}{d}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] - B_m^2 \qquad (8)$$

and a similar procedure employed to calculate C_H at each of the vertical measurement positions.

All of these calculations are iterative in that the higher harmonics are first neglected in calculating γ_{mn}^2 from (6) or (8). Preliminary values for C_H at each measurement position are thus obtained and applied to the data. This in turn yields a new γ_{11} and the procedure is repeated until successive approximations cause no change in the slope. It is necessary to conduct these calculations concurrently with

the iteration for end leakage. It was found that accuracy within the limits of counting statistics was generally obtained by (1) fitting the slope by eye (2) applying corrections (3) fitting altered slope by eye (4) applying corrections, and finally (5) fitting the slope by the method of weighted least-squares.

The same equipment was used for all BF_3 measurements reported in this thesis. High voltage was applied to the detector tube approximately 48 hours before any counts were made. Counter-scaler settings were not changed during the entire experiment. Data were not corrected for deviation from standard temperature and pressure, since such corrections are variable and extremely small. An adequate check upon counting stability was provided by periodic measurements at several standard positions. No statistically valid trend could be observed; hence no stability compensation was applied. The data are reported in Table VIII of Appendix I.

The neutron sources used are 2.54 x 3.65 cm Pu-Be cylinders clad in tantalum and stainless steel. Each of the two sources contains one curie of Pu^{239} and emits approximately 1.6×10^6 fast neutrons per second. Average energy of the source spectrum is 4 Mev (S-3). The chamber of the Nancy-Wood BF_3 detector used is 10 cm long, 1.59 cm in diameter, has an active length of 8.25 cm, and is filled to a pressure of 40 mm of Hg with BF_3 enriched in boron-10. This counter was connected directly to a binary scaling circuit.

D. BF_3 Measurements -- Pile Unloaded

After all uranium had been removed from the pile, a further series of measurements was made to determine neutron diffusion length in both the conventional and reciprocal arrangements. First the usual sigma pile experiment was conducted, with source in the pedestal and detector traverses made through the same seventeen stations in the unloaded lattice. This again included both bare and cadmium covered measurements. Pile background was very low and essentially independent of position; thus it was found that single background counts with and without cadmium cover, were sufficient to define thermal neutron background throughout the unloaded pile.

Reciprocal measurement was accomplished by reversing source and detector and repeating the conventional traverses. Background was particularly low in this configuration, since the detector remains within a well shielded pedestal.

Again an outline of conditions under which BF_3 counts were made in the unloaded pile is given; Figure 3 and Table I of Appendix I specify stations. (Discontinuity of run numbers is due to the fact that runs were numbered chronologically).

Run 12 Source in pedestal: count with detector at stations 1 - 11 and a - g, with and without cadmium covers in place. (34 counts)

Run 13 No source in pile: count with detector at stations 1 - 11 and a - g, with and without cadmium covers in place, to determine thermal neutron background. (34 counts - found statistically constant)

Run 14 Detector in pedestal: count with source in stations 1 - 11 and a - g, with and without cadmium covers in place. (34 counts)

Run 15 Detector in pedestal: background count with no source in pile, with and without cadmium cover in place. (2 counts)

All comments of section II. C. regarding statistics, corrections, and equipment apply equally to these unloaded measurements.

E. BF_3 Measurements - Shutters Installed

The BF_3 data mentioned during discussion of loaded and unloaded measurements were designed to provide experimental values of pile buckling and diffusion length. Moreover, the methods employed furnished a direct evaluation of reciprocal methods.

It was felt that additional measurements might provide a better insight into the importance function. Specifically, flux determinations were made with both source and detector within the loaded lattice. Cadmium shutters were fabricated from sheet stock and inserted into slots provided at the lattice-pedestal interface. Reflection of thermal neutrons from the pedestal was thus minimized, and lattice boundary conditions were made more nearly symmetric. A total of twelve complete traverses were conducted to measure thermal and epithermal neutron density. These consisted of alternately positioning bare or cadmium covered source and detector about the central and two extreme vertical measurement positions.

BF_3 counts made with shutters installed are here outlined, specifying conditions of measurement.

Run 5	Cadmium shutters installed, source in station 6(f): count with detector at stations 1 - 5 and 7 - 11, with and without cadmium covers in place. (20 counts)
Run 6	Cadmium shutters installed, detector in station 6(f): count with source in stations 1 - 5 and 7 - 11, with and without cadmium covers in place. (20 counts)
Run 7	Cadmium shutters installed, no source in assembly: count with detector in each of stations 1 - 11 and a - g, with and without cadmium covers in place. (34 counts). This is similar to Run 2, but with cadmium shutters installed.
Run 8	Cadmium shutters installed, source in station 1: count with detector at stations 2 - 11 and a - g, with and without cadmium covers in place. (32 counts)
Run 9	Cadmium shutters installed, detector in station 1: count with source in stations 2 - 11 and a - g, with and without cadmium covers in place. (32 counts)
Run 10	Cadmium shutters installed, source in station 11: count with detector in stations 1 - 10 and a - g, with and without cadmium covers in place. (32 counts)
Run 11	Cadmium shutters installed, detector in station 11: count with source in stations 1 - 10 and a - g, with and without cadmium covers in place. (32 counts)

Data from runs 5 - 11 appear in Table VII, Appendix I

Previous mention of statistical accuracy and resolving time is applicable to these data. Further corrections were not applied.

III. RESULTS

A. Foil Measurements

Data obtained through indium foil measurements did not contribute to the information required of this experiment. Statistical errors prohibited accurate calculation of material buckling of diffusion length. This factor combined with insufficient data in the slowing down region to prevent experimental determination of Fermi Age. Consequently the data are reported in Table IV, but no further calculations are made.

B. BF_3 Measurements -- Pile Loaded

The primary purpose of these measurements was to provide experimental values for material buckling obtained by conventional and reciprocal means. Correlation of results for these two methods was excellent, strongly indicating the validity of reciprocal data. Certain very definite advantages are associated with the novel procedures proposed in this thesis, particularly with regard to statistical accuracy and precision. All data for these measurements are included in Table V.

1. Horizontal Distribution

Figure 5 shows the horizontal flux distribution at mid-height under both conventional and reciprocal conditions. Plotted points indicate experimental results and standard error in the data. These are referenced

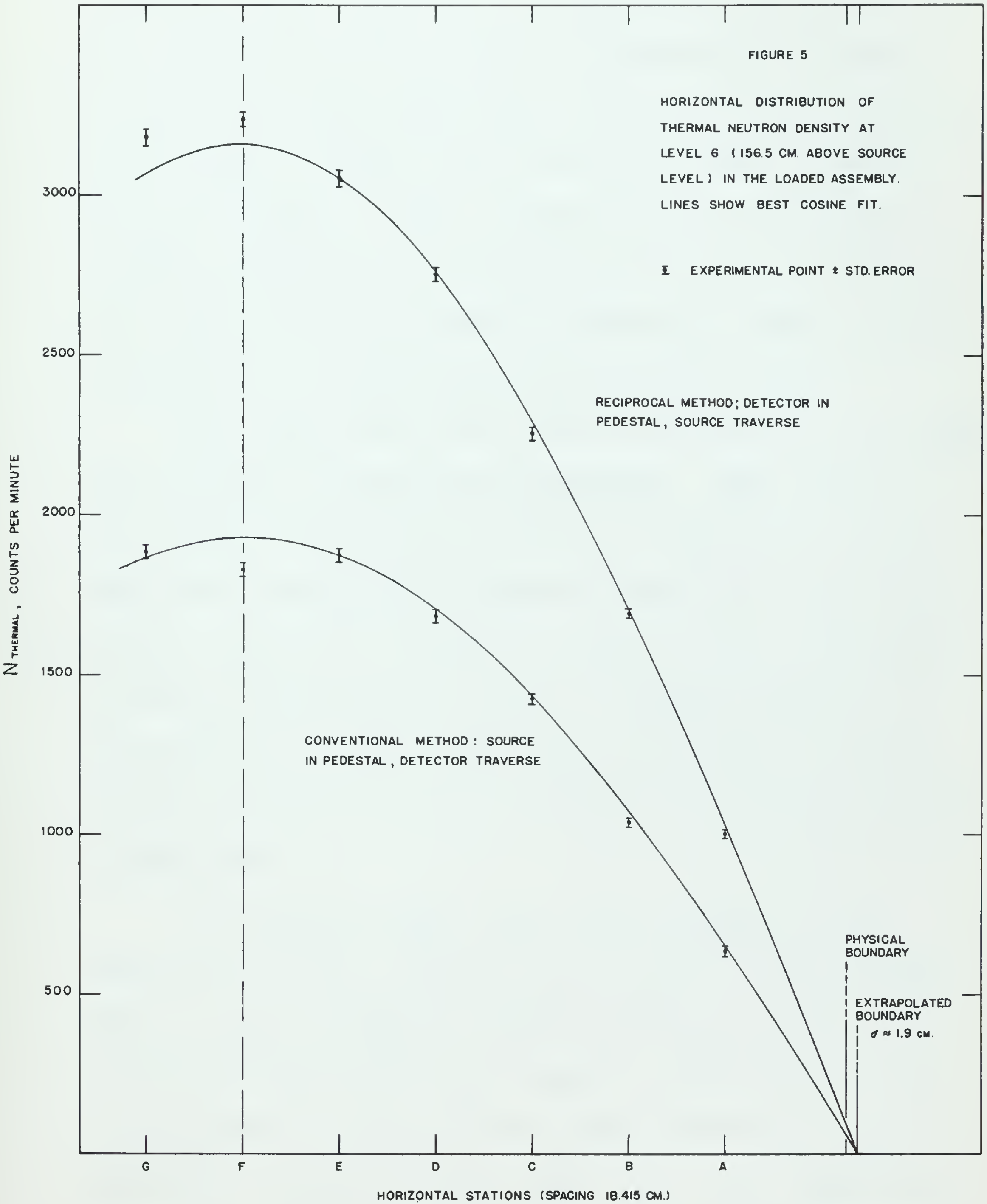


FIGURE 5
HORIZONTAL DISTRIBUTION OF THERMAL NEUTRON DENSITY AT LEVEL 6 (156.5 CM. ABOVE SOURCE LEVEL) IN THE LOADED ASSEMBLY. LINES SHOW BEST COSINE FIT.
EXPERIMENTAL POINT ± STD. ERROR
RECIPROCAL METHOD; DETECTOR IN PEDESTAL, SOURCE TRAVERSE
CONVENTIONAL METHOD: SOURCE IN PEDESTAL, DETECTOR TRAVERSE
PHYSICAL BOUNDARY
EXTRAPOLATED BOUNDARY
d ≈ 1.9 CM.
3000
2500
2000
1500
1000
500
N THERMAL, COUNTS PER MINUTE
G
F
E
D
C
B
A
HORIZONTAL STATIONS (SPACING 18.415 CM.)

to a cosine distribution, which was normalized by the method of least-squares. A reasonably good cosine fit was obtained in both cases. Mathematical fitting of these curves was undertaken, since they were used quantitatively to deduce the horizontal linear extrapolation distance. Both conventional and reciprocal curves converge to an extrapolated boundary 0.75" (1.9 cm) beyond the physical dimension, which is characteristic of other experimental values (K-1).

This value was also used in calculations involving the vertical extrapolation distance. More properly, the extrapolated height should be obtained by trial and error, minimizing squared deviations of the experimental points for different values of this dimension. This is prohibitively tedious if manual methods of calculation must be used, since the process becomes doubly iterative with the fitting for end and harmonic corrections.

It should be noted that any reasonable curve drawn through the conventional data has a pronounced dip or flux depression between stations "g" and "e", centered approximately on the pile centerline. Pile loading was uniform in this region and there are no geometric irregularities in the pile which might contribute to this effect. The phenomenon was observed during loaded and unloaded traverses made in the conventional manner and has not been satisfactorily explained.

2. Vertical Distribution

Results of conventional and reciprocal traverses made in the vertical direction are shown in Figure 6. All data points were corrected for diffusion harmonics and end leakage. Standard error is indicated by brackets on the individual points. Fitting was by the method of least-squares for stations 3 through 11, points being weighted inversely with their variance. The slopes of these lines, and hence the pile bucklings, are identical within statistical error.

Certain anomalies are apparent in the results, and these are discussed more thoroughly in Appendix IV. A qualitative physical explanation for the large difference in counting rate has been developed. Furthermore, as may be seen from Figure 6, conventional data for stations where source and detector are in closest proximity exhibit an appreciable effect due to slowing down density and change in energy distribution. This effect is absent in the reciprocal configuration, where inclusion of Stations 1 and 2 in a weighted least-squares analysis does not alter the results. Appendix II shows that these two reciprocal measurements are statistically acceptable.

Material buckling was experimentally obtained by each method. These values are:

Conventional	Reciprocal
$B_m^2 = 86 \pm 12 \times 10^{-6}\ cm^{-2}$	$B_m^2 = 80 \pm 11 \times 10^{-6}\ cm^{-2}$

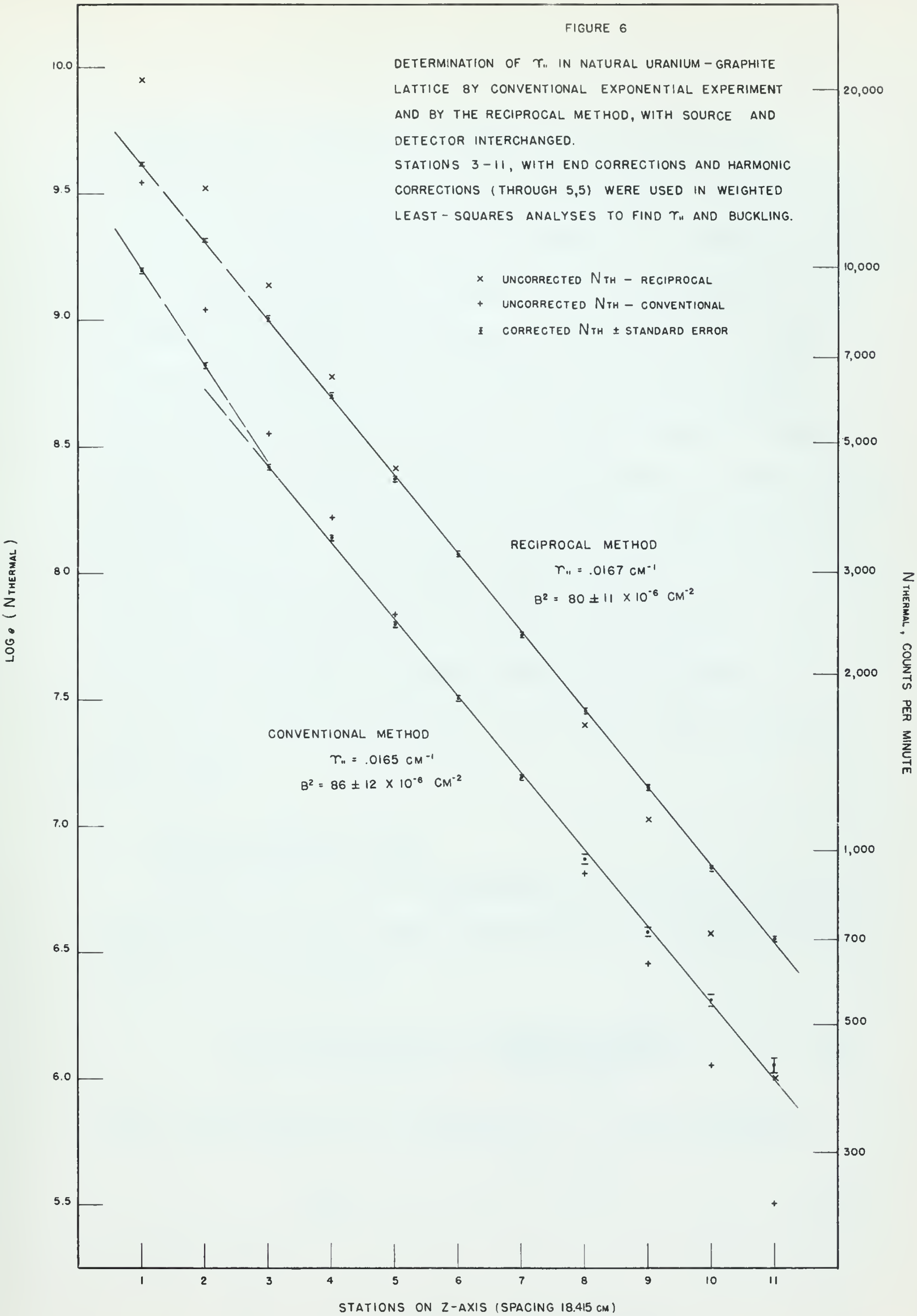


FIGURE 6
DETERMINATION OF Υ_{11} IN NATURAL URANIUM – GRAPHITE LATTICE BY CONVENTIONAL EXPONENTIAL EXPERIMENT AND BY THE RECIPROCAL METHOD, WITH SOURCE AND DETECTOR INTERCHANGED.
STATIONS 3 – 11, WITH END CORRECTIONS AND HARMONIC CORRECTIONS (THROUGH 5,5) WERE USED IN WEIGHTED LEAST – SQUARES ANALYSES TO FIND Υ_{11} AND BUCKLING.
× UNCORRECTED N_{TH} – RECIPROCAL
+ UNCORRECTED N_{TH} – CONVENTIONAL
CORRECTED N_{TH} ± STANDARD ERROR
RECIPROCAL METHOD
Υ_{11} = .0167 CM^{-1}
$B^2 = 80 \pm 11 \times 10^{-6}$ CM^{-2}
CONVENTIONAL METHOD
Υ_{11} = .0165 CM^{-1}
$B^2 = 86 \pm 12 \times 10^{-6}$ CM^{-2}
LOG$_e$ ($N_{THERMAL}$)
10.0
9.5
9.0
8.5
8.0
7.5
7.0
6.5
6.0
5.5
$N_{THERMAL}$, COUNTS PER MINUTE
20,000
10,000
7,000
5,000
3,000
2,000
1,000
700
500
300
1
2
3
4
5
6
7
8
9
10
11
STATIONS ON Z-AXIS (SPACING 18.415 CM)

A superficial inspection would indicate that the standard error reported for these results is excessively high, considering the 1% accuracy in counting statistics which was maintained throughout the experimental work. The error reported is appropriate,resulting in part from the propagation of counting error but largely from a small difference in the subtraction of large numbers. This may be readily visualized by examination of equation (8), from which the results are obtained. Valid comparison with other experimenters is difficult because of differences in lattice arrangement. For lattices of similar design, material bucklings of about 55×10^{-6} to 88×10^{-6} cm^{-2} have been obtained (D-1, G-5).

3. Corrections to Data

Referring to the development on pages 17 and 18, experimental points were corrected to the fundamental mode by the relation

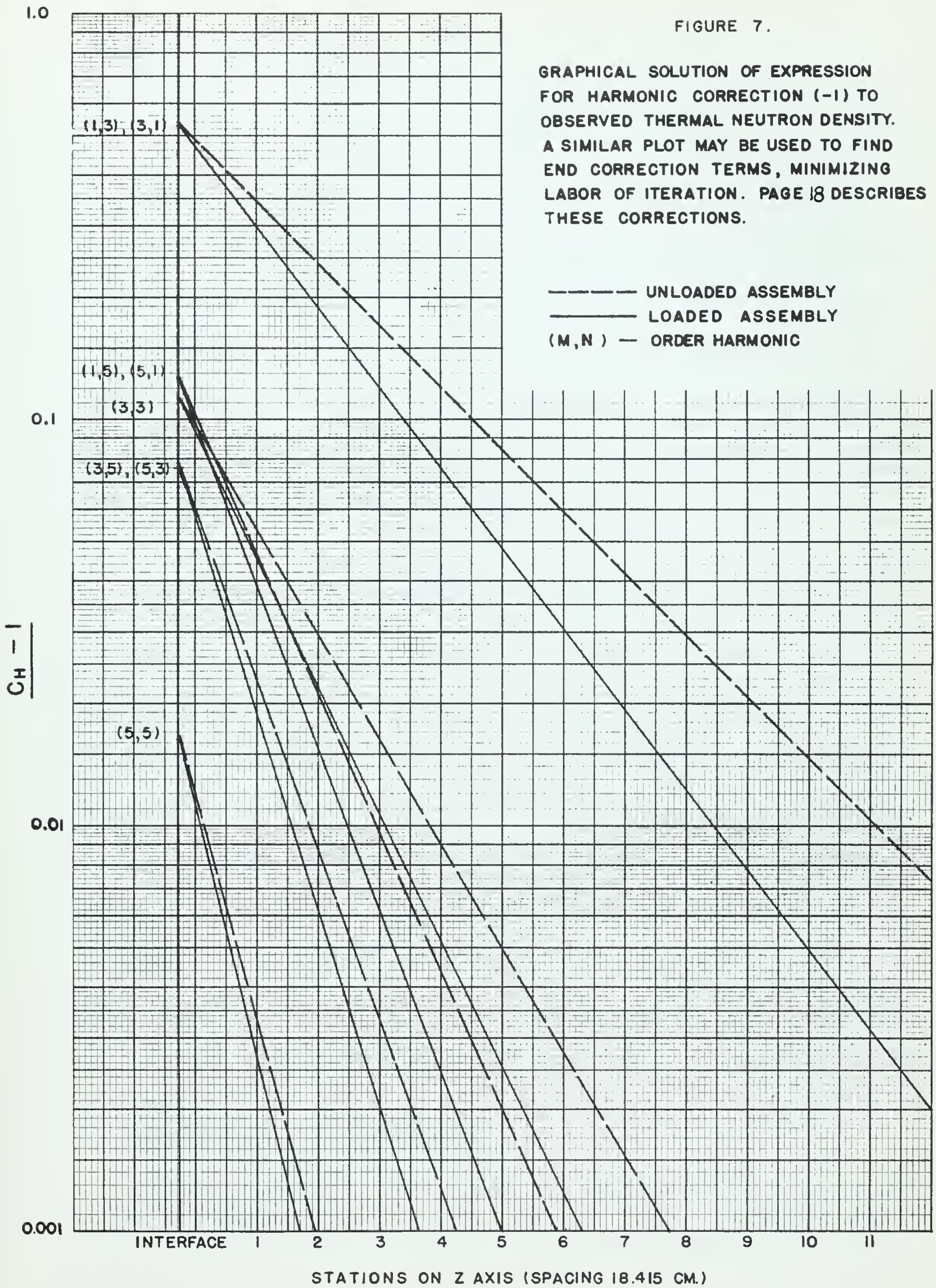
$$N_{th}(\text{fundamental}) = \frac{N_{th}(\text{measured})}{C_H \, C_E} \tag{9}$$

where end leakage is compensated by the expression

$$C_E = 1 - e^{-2\gamma(c-z)} \tag{10}$$

and the effect of diffusion harmonics is corrected by

$$C_H = 1 + \left[\frac{\gamma_{11}}{\gamma_{mn}} e^{-(\gamma_{mn} - \gamma_{11})z}\right] \tag{7}$$



1.0
0.1
0.01
0.001
FIGURE 7.
GRAPHICAL SOLUTION OF EXPRESSION FOR HARMONIC CORRECTION (-1) TO OBSERVED THERMAL NEUTRON DENSITY. A SIMILAR PLOT MAY BE USED TO FIND END CORRECTION TERMS, MINIMIZING LABOR OF ITERATION. PAGE 18 DESCRIBES THESE CORRECTIONS.
UNLOADED ASSEMBLY
LOADED ASSEMBLY
(M,N) — ORDER HARMONIC
(1,3), (3,1)
(1,5), (5,1)
(3,3)
(3,5), (5,3)
(5,5)
$C_H - 1$
INTERFACE
1
2
3
4
5
6
7
8
9
10
11
STATIONS ON Z AXIS (SPACING 18.415 CM.)

where

$$\gamma_{mn}^2 = \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \right] - B_m^2 \qquad (8)$$

In these calculations, γ_{11} is the reciprocal relaxation distance of the fundamental harmonic; B_m^2 is the material buckling. The above expressions are standard for such corrections. Harmonic corrections were made on a two region basis through the 5th harmonic. This consisted of calculating an attenuation of each harmonic within the pedestal region from (6) employing

$$\gamma_{mn}^2 = \kappa^2 + \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \right] \qquad (6)$$

Having the relative harmonic strengths at the lattice-pedestal interface, it was a relatively easy matter to attenuate these harmonics exponentially according to equations (6) and (7). This was done graphically on the semi-logarithmic plot of Figure 7, saving much labor of calculation and sacrificing no meaningful accuracy. It should be noted that end and harmonic corrections are by no means negligible at any position in a small assembly employing a point source of neutrons.

4. Evaluation of Technique

The statistical benefits of reciprocal measurements have already been mentioned and were graphically shown in Figures 5 and 6. For any desired degree of accuracy, actual counting times in a given position are reduced

over 25% by virtue of the greater counting rates obtained in the reciprocal configuration. The precision of reciprocal measurements is far superior, as discussed in Appendix II.

Handling of the source can be accomplished safely and effectively without elaborate equipment, though the procedure requires respect and appreciation for possible danger involved. During the four week period of thesis experimentation, ionizing radiation received by the participants was never significantly above background. The average accumulated neutron dosage was 20 millirem, as measured by tracks counted on a fast neutron film. This dosage, again, was not significant.

An absence of detector handling, on the other hand, presents decided advantages. Permitting the BF_3 tube to remain in a single position throughout an experiment promises better electronic stability, less likelihood of damage to the tube, and a probability of more consistent results.

Calculation procedures are considerably abbreviated for reciprocal data, since pedestal background is measured once and remains constant for all source positions. Since fewer measurements are required and higher counting rates are obtained in the reciprocal arrangement, the overall time required for experimental work is approximately 40% that of a conventional experiment.

C. BF_3 Measurements - Pile Unloaded

After all fuel was removed from the exponential assembly, measurements were made to permit determination of neutron diffusion length in the graphite used in this assembly. These data were obtained by conventional and reciprocal experiments, and correlation of results was again excellent. Although reciprocity was thus shown to be valid in an unloaded assembly, certain of the advantages applying to the reciprocal method in an exponential experiment were of less consequence in this phase of the investigation.

1. Horizontal Distribution

Horizontal measurements of neutron density by reciprocal and conventional methods are shown in Figure 8. These data permit an acceptable fit with an approximately normalized cosine distribution. Linear extrapolation distance for horizontal flux was determined to be 0.75" (1.9 cm) -- identical to the result obtained in a loaded lattice. This value was again used for vertical extrapolation, for reasons previously discussed. The conventional data indicate a central flux depression similar to that observed during measurements performed in the loaded pile.

2. Vertical Distribution.

Figure 9 includes the results of conventional and reciprocal traverses along the z-axis. Measurements

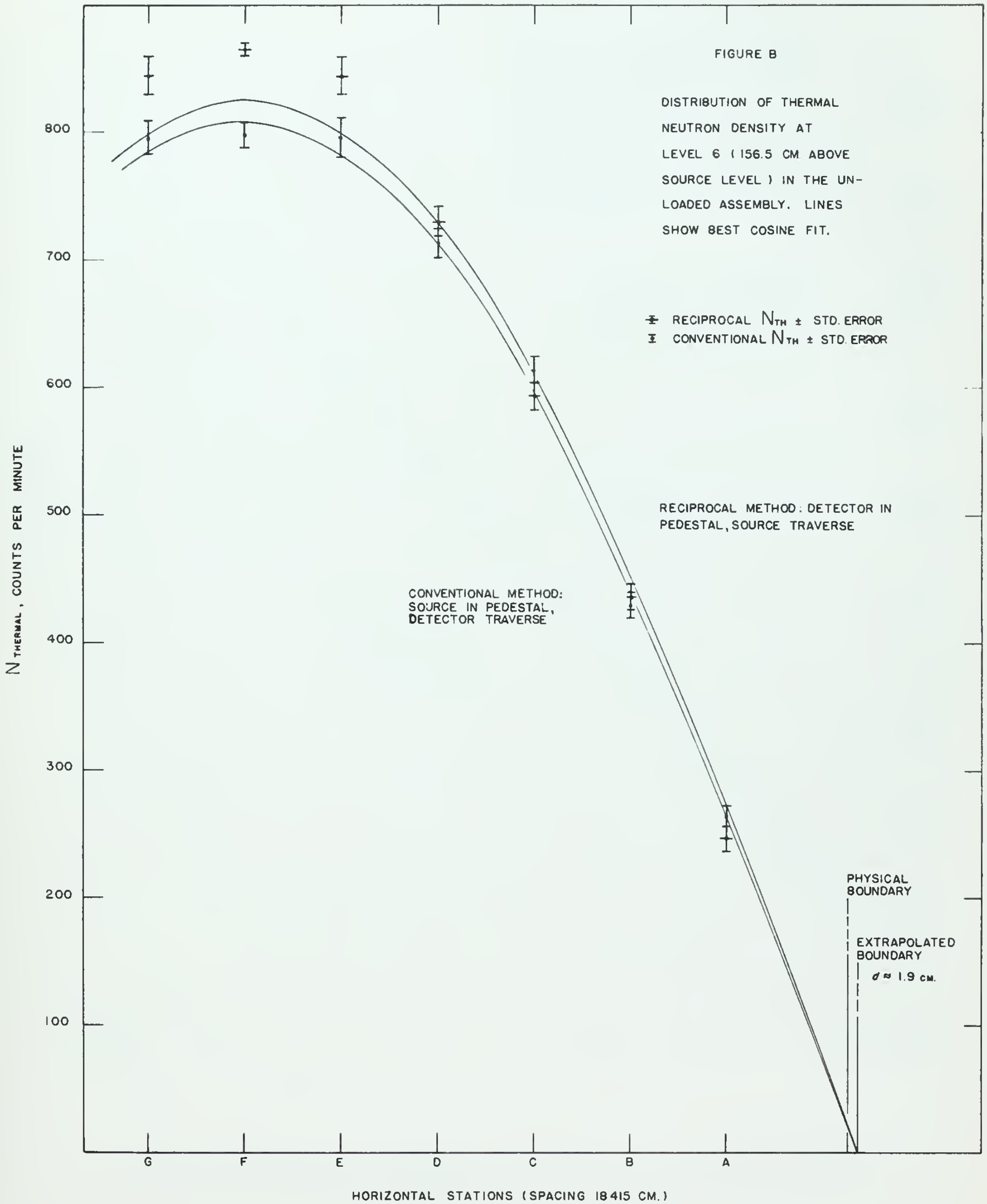


FIGURE B
DISTRIBUTION OF THERMAL NEUTRON DENSITY AT LEVEL 6 (156.5 CM ABOVE SOURCE LEVEL) IN THE UN-LOADED ASSEMBLY. LINES SHOW BEST COSINE FIT.
RECIPROCAL N_{TH} ± STD. ERROR
CONVENTIONAL N_{TH} ± STD. ERROR
RECIPROCAL METHOD: DETECTOR IN PEDESTAL, SOURCE TRAVERSE
CONVENTIONAL METHOD: SOURCE IN PEDESTAL, DETECTOR TRAVERSE
PHYSICAL BOUNDARY
EXTRAPOLATED BOUNDARY
$d \approx 1.9$ CM.
$N_{THERMAL}$, COUNTS PER MINUTE
800
700
600
500
400
300
200
100
G
F
E
D
C
B
A
HORIZONTAL STATIONS (SPACING 18.415 CM.)

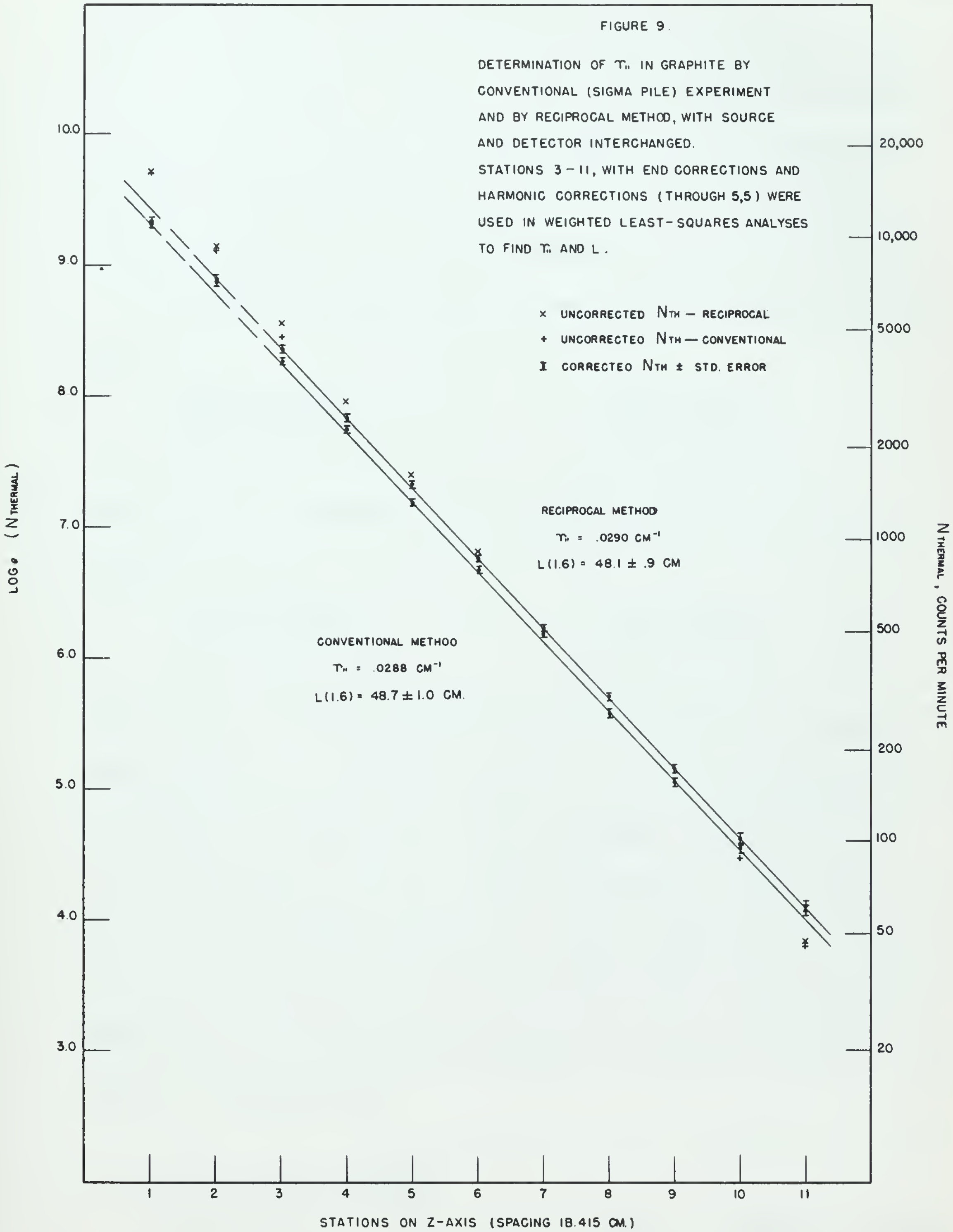


FIGURE 9.
DETERMINATION OF τ_{th} IN GRAPHITE BY CONVENTIONAL (SIGMA PILE) EXPERIMENT AND BY RECIPROCAL METHOD, WITH SOURCE AND DETECTOR INTERCHANGED.
STATIONS 3 - 11, WITH END CORRECTIONS AND HARMONIC CORRECTIONS (THROUGH 5,5) WERE USED IN WEIGHTED LEAST-SQUARES ANALYSES TO FIND τ_{th} AND L.
× UNCORRECTED N_{TH} — RECIPROCAL
+ UNCORRECTED N_{TH} — CONVENTIONAL
I CORRECTED N_{TH} ± STD. ERROR
RECIPROCAL METHOD
τ_{th} = .0290 CM^{-1}
L(1.6) = 48.1 ± .9 CM
CONVENTIONAL METHOD
τ_{th} = .0288 CM^{-1}
L(1.6) = 48.7 ± 1.0 CM.
LOG_e ($N_{THERMAL}$)
$N_{THERMAL}$, COUNTS PER MINUTE
STATIONS ON Z-AXIS (SPACING 18.415 CM.)
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
20,000
10,000
5000
2000
1000
500
200
100
50
20
1
2
3
4
5
6
7
8
9
10
11

were corrected for end effect in the usual manner. Two region harmonic corrections were again applied as previously outlined. The need for a two region analysis is much less for the unloaded data, since relaxation length differs between regions only because of the density change due to air channels within the lattice.

Diffusion length as determined by the two methods is:

Conventional	Reciprocal
$L = 46.1 \pm 0.9$ cm	$L = 45.5 \pm 0.8$ cm

Corrected to standard (1.6 gm/cm^3) graphite, these results become:

Conventional	Reciprocal
$L = 48.7 \pm 1.0$ cm	$L = 48.1 \pm 0.9$ cm

These values are somewhat below a world-consistent measurement of 52.0 ± 1.0 cm (K-1). The discrepancy is attributed to impurity of the surplus graphite used in construction, and to other impurities which have accumulated within the pile. More important to the purposes of this thesis is the fact that statistically identical results were obtained by the two methods, as shown in Appendix II.

3. Corrections to Data

Equations (6), (7), (9) and (10) of Section IV.B.3. were used to correct experimental measurements for diffusion harmonics and end leakage. This process, which

again employed graphical methods, requires no further discussion. Figure 7 illustrates harmonic effects.

Corrections to compensate for cylindrical voids in the moderating medium were investigated. According to Wilson, et al (W-3), the ratio of diffusion length parallel ($L_{\parallel}$) and perpendicular ($L_{\perp}$) to a channel axis is given by:

$$\left(\frac{L_{\parallel}}{L_{\perp}}\right)^2 = \frac{L_0^2\left[1 + 2\psi(1 + r/\lambda_{tr})\right]}{L_0^2\left[1 + \psi(2 + r/\lambda_{tr})\right]} \tag{11}$$

where ψ is the ratio of void volume to moderator volume, r is the void hydraulic radius, and λ_{tr} is the transport mean free path in a homogenized lattice. Thence the expression for diffusion length becomes:

$$\frac{1}{L_{\perp}^2} = \kappa_{\perp}^2 = \gamma_{11\perp}^2 - \left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2\left(\frac{L_{\parallel}}{L_{\perp}}\right)^2\right] \tag{12}$$

It was determined that this correction is totally negligible in the M.I.T. assembly.

For comparison with published data it was necessary to correct the experimental diffusion length to standard density, temperature, and pressure. This was done by the method presented by Wilson, et al (W-3):

$$L^2(1.6) = L^2\left[1 + (1/\rho_1 - 1/\rho_0)\frac{\sigma_a}{\sigma_c}\cdot\frac{P}{T}\cdot 1.925\cdot 10^{-4}\right](\rho_2/1.6)^2 \tag{13}$$

where: P = atmospheric pressure in mm of Hg
T = graphite temperature in °K
ρ_0 = density of crystalline graphite
ρ_1 = average density of the lattice including empty channels
ρ_2 = average density of solid graphite in the lattice

σ_a = weighted average neutron cross section of air (1.474 barns)
σ_c = cross section for pure carbon (.003748 barns)

It should again be noted that harmonic effects and end leakage are of consequence at all positions of measurement.

4. Evaluation of Technique

Figures 8 and 9 indicate a slight advantage in counting rate associated with the reciprocal method. This has been attributed to geometrical differences and is fully discussed in Appendix IV. Benefits of electronic stability mentioned in evaluation of exponential techniques apply equally to the unloaded measurements. In summary, it may be said that reciprocal methods are completely acceptable for the sigma pile experiment and that a considerable improvement in precision of measurement is gained. The effect of experimental technique upon precision is discussed more fully in Appendix II.

C. BF_3 Measurements -- Pile Loaded, Shutters Installed

These measurements, originally designed to yield additional insight regarding behavior of the importance function, consisted of several series of flux traverses under varying conditions of source-detector orientation. In all cases, lattice and pedestal were isolated by cadmium shutters installed at the interface. Measurements by conventional and

reciprocal methods defined the thermal flux employing stations 1, 6(f), and 11 as reference points.

The data thus obtained has been corrected to relative thermal density and qualitatively examined. The validity of reciprocity and the adjoint flux concept was established by one-group analyses of experiments conducted without the cadmium shutters in place; no reason to modify previous conclusions was inferred from the shutter data.

It is possible that a more thorough analysis might provide information which would yield better definition of boundary conditions, reflector effect, and fast fission. Such a study is beyond the scope of this thesis.

IV. CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

1. Measurement of pile parameters in an exponential assembly containing a single source may be performed equally well when source and detector are interchanged.
2. Corrections to reciprocal data for end leakage and harmonic effect may be applied in the usual manner. The methods for obtaining these corrections remain unchanged.
3. Accuracy of measurement is considerably improved by use of reciprocal methods, and somewhat fewer measurements are required to define the flux distribution. Hence the time required for experimental work is reduced appreciably.
4. Calculations required to reduce reciprocal data to thermal density are shortened, diminishing the time required for computation of parameters.
5. All points of measurement are equally valid in the novel experiment.
6. The precision of reciprocal measurements is significantly greater than that obtained by conventional methods.
7. No appreciable radiation hazard is associated with reciprocal techniques, provided reasonable precautions are observed.

8. Information regarding neutron energy distribution in a loaded lattice may be obtained by comparison of conventional and reciprocal experiments. This possibility is discussed in Appendix IV.

B. <u>Recommendations</u>

1. Educational and research institutions should consider the reciprocal technique as a possible improvement in the traditional method of performing an exponential experiment. A presentation of the physical meaning of importance is implicit in the novel approach.
2. Theoretical calculations and experimental work should be performed to verify or disprove the validity of measurements made with four detectors in the pedestal and a single source in the lattice. This arrangement should provide cancellation of certain major harmonics.
3. Further investigation of the relevance of this experiment to definition of neutron spectra is recommended. Appendix IV contains some ideas in relation to this aspect of the experiment.

APPENDICES

APPENDIX I

TABLES OF DATA

The following tables present all experimental data obtained during the experimental phases of this thesis. Certain further information is included relating to physical and nuclear characteristics of the M.I.T. exponential assembly.

All measurements of neutron density have been corrected to the extent indicated by notes following the individual data.

TABLE I

Coordinates of Foil Measurement Positions

Position	x	y	z
11	-4.8125	-1.75	83.25
10	-4.8125		76
9	-4.8125		68.75
8	-4.8125		61.50
7	-4.8125		54.25
6	-4.8125		47
g	-4.8125		47
f	-10.6250		47
e	-16.4375		47
d	-22.2500		47
c	-28.0625		47
b	-33.8750		47
a	-39.6875		47
5	-4.8125		39.75
4	-4.8125		32.50
3	-4.8125		25.25
2	-4.8125		18
1	-4.8125		10.75
S	-1.6250	0	-16.25

Notes:

(1) $z = 0$ at lattice-pedestal interface

(2) All coordinates expressed in inches from intersection of pile centerline and $z = 0$ plane.

TABLE I (Continued)

Coordinates of BF_3 Measurement Positions

Position	x	y	z
11	0	-1.75	81.625
10	0		74.375
9	0		67.125
8	0		59.875
7	0		52.625
6	0		45.375
g	-7.250		45.375
f	0		45.375
e	7.250		45.375
d	14.500		45.375
c	21.750		45.375
b	29		45.375
a	36.250		45.375
5	0		38.125
4	0		30.875
3	0		23.625
2	0		16.375
1	0		9.125
S	-1.625	0	-16.250

Notes:

(1) $z = 0$ at lattice-pedestal interface

(2) All coordinates expressed in inches from intersection of pile centerline and $z = 0$ plane.

TABLE II

SUMMARY OF PHYSICAL CHARACTERISTICS OF EXPONENTIAL PILE

A. Linear Dimensions

x	90.75"	(east-west)
y	91.00"	(north-south)
z	90.75"	(vertical-lattice only)

B. Volumes

Overall volume, including air channels	749,436 in^3
Volume of air channels	17,205 in^3
Volume of graphite (V_G)	732,231 in^3
Volume of fuel (V_U)	8,099 in^3

$$\frac{V_G}{V_U} = 90.41$$

C. Fuel

Uniform loading	9 slugs per fuel channel
No. channels	144
Total slugs*	1289
Wt. per slug	4.275 lb
Slug dimensions	1.087" OD
	8.375" length
atoms/cm^3 (N_U)	4.7325×10^{22}

D. Moderator

Density	1.662 gm/cm^3
atoms/cm^3 (N_G)	8.3370×10^{22}
$N_G/N_U \times V_G/V_U$	159.27
$N_U/N_G \times V_U/V_G$	.006279

*Loading in isolated channels departed from the uniform loading.

TABLE III

SUMMARY OF MEASURED PILE PARAMETERS

		Conventional	Reciprocal
A.	Material Buckling	$86 \pm 12 \times 10^{-6} cm^{-2}$	$80 \pm 11 \times 10^{-6} cm^{-2}$
B.	Diffusion Length (unloaded)	46.1±.9cm	45.5 ±.8cm
C.	Diffusion Length (unloaded) (corrected to std. conditions)	48.7 ± 1.0cm	48.1 ±.9cm
D.	Diffusion Length (loaded)*	14.71 cm	14.54 cm
E.	Fermi Age*	354 cm^2	354 cm^2
F.	Infinite Multiplication Factor (K_∞)*	1.049	1.045

*Calculated value

TABLE IV

Indium Foil Data

Foil Posit	N_B	N_C	N_b	CR	N_{th}
11	21.33	6.71	13.10	3.18	4.97
10	43.81	13.50	13.27	3.25	20.55
9	64.61	17.71	17.84	3.69	33.95
8	104.66	20.17	21.16	5.19	68.90
7	132.48	28.01	17.23	4.73	91.77
6	144.59	41.07	20.58	3.52	87.36
g	144.59	41.07	20.58	3.52	87.36
f	147.03	35.81	23.67	4.11	83.79
e	153.69	28.69	20.69	5.36	109.76
d	110.33	26.40	18.69	4.18	70.16
c	96.46	20.76	20.44	4.65	60.63
b	74.14	15.16	21.81	4.89	42.89
a	34.56	11.21	22.06	3.08	7.09
5	203.28	50.89	19.87	3.99	137.73
4	302.40	53.50	23.27	5.65	231.65
3	434.12	67.56	17.98	6.43	353.31
2	604.50	124.50	17.38	4.86	467.20
1	937.63	129.13	21.70	7.26	782.42

These data do not include harmonic or end corrections. The number of significant figures reported is a result of calculation methods employed and is by no means a measure of statistical accuracy.

TABLE V

BF_3 Data - Pile Loaded

Conventional (Source in pedestal, detector in position)

Posit	N_B	σ_B	N_C	σ_C	N_b	σ_b	N_c	σ_c	N_{th}	σ_{th}	N_{th} Corr
11	304	6	12	1	45	2	2	1	247	6	429
10	518	7	18	1	74	3	3	1	428	8	552
9	755	9	27	2	92	3	4	1	639	9	722
8	1052	10	38	2	104	3	4	1	914	11	965
7	1465	14	56	2	107	3	5	1	1308	14	1328
6	2027	18	73	1	118	3	5	1	1842	19	1817
g	2078	19	72	3	120	4	5	1	1890	19	1881
f	2027	18	73	1	118	3	5	1	1842	19	1817
e	2065	19	69	3	119	3	5	1	1883	19	1874
d	1860	18	65	3	112	3	4	1	1687	18	1684
c	1574	16	54	2	102	3	5	1	1422	17	1423
b	1143	12	33	2	78	3	5	1	1034	12	1039
a	695	8	24	2	49	2	2	1	624	9	630
5	2738	23	101	1	112	3	5	1	2529	24	2426
4	3960	36	139	1	115	2	6	1	3713	36	3439
3	5463	52	191	1	107	3	5	1	5170	52	4545
2	8786	66	285	2	100	3	3	1	8404	66	6796
1	14,416	120	399	2	92	3	2	1	13,927	120	9881

Reciprocal (Detector in pedestal, source in position)

Posit											
11	442	5	3	1	33	0	0	0	407	5	708
10	756	9	3	1					720	9	928
9	1170	12	8	1					1129	12	1275
8	1684	18	9	1					1642	18	1734
7	2336	22	12	1					2291	22	2325
6	3313	24	20	1					3260	24	3216
g	3250	23	18	1					3199	23	3183
f	3313	24	20	1					3260	24	3216
e	3120	23	18	1					3069	23	3054
d	2799	24	15	1					2751	24	2746
c	2297	21	11	1					2253	21	2255
b	1723	19	11	1					1679	19	1687
a	1032	10	8	1					991	10	1001
5	4579	48	28	2					4518	48	4334
4	6562	57	39	2					6490	57	6011
3	9411	97	59	2					9319	97	8192
2	13,817	117	97	3					13,687	117	11,068
1	21,277	146	188	4					21,056	146	14,939

TABLE V

Notes:

(1) Resolving time corrections have been applied to N_B when appropriate.

(2) N_{th} (corrected) includes corrections for diffusion harmonics and end leakage

(3) All data is rounded to the nearest whole count. This was done after the best values had been finally corrected.

TABLE VI

BF$_3$ Data - Pile Unloaded

Conventional (Source in pedestal, detector in position)

Posit	N_B	σ_B	N_C	N_{th}	σ_{th}	N_{th} Corr	M
11	46	2	1	45	2	59	5.49
10	92	3	3	89	3	96	4.81
9	158	4	0	158	4	159	4.05
8	271	5	1	270	5	265	3.38
7	512	7	3	509	7	489	2.57
6	851	9	3	848	9	797	2.17
g	845	13	2	843	13	795	2.24
f	851	9	3	848	9	797	2.17
e	847	13	3	844	13	795	2.23
d	758	12	4	754	12	713	2.24
c	647	11	1	646	11	615	2.20
b	450	10	1	449	10	429	2.30
a	276	8	1	275	8	265	2.27
5	1459	14	4	1455	14	1332	1.74
4	2635	23	8	2627	23	2314	1.41
3	4688	40	19	4669	40	3916	1.11
2	9197	68	52	9145	68	7015	.92
1	16,558	129	108	16,450	129	11,182	.85

Reciprocal (Detector in pedestal, source in position)

Posit							
11	47	2	0	47	2	61	8.66
10	95	3	0	95	3	102	7.58
9	175	4	0	175	4	176	6.45
8	309	6	2	307	6	301	5.35
7	533	7	3	530	7	509	4.35
6	923	4	3	920	4	865	3.54
g	901	13	7	894	13	843	3.58
f	923	4	3	920	4	865	3.54
e	898	13	4	894	13	843	3.43
d	775	12	3	772	12	730	3.56
c	626	11	2	624	11	594	3.61
b	457	10	1	456	10	436	3.68
a	255	7	0	255	7	246	3.89
5	1667	15	6	1661	15	1521	2.72
4	2907	24	11	2896	24	2551	2.24
3	5162	42	18	5144	42	4287	1.81
2	9413	69	48	9365	69	7184	1.46
1	16,789	129	134	16,654	129	11,321	1.26

TABLE VI

Notes:

(1) Resolving time corrections have been applied to N_B when appropriate.

(2) N_{th} (corrected) includes corrections for diffusion harmonics and end leakage

(3) All data is rounded to the nearest whole count. This was done after the best values had been finally corrected.

(4) The standard deviation of N_C was not statistically significant.

(5) N_B, as reported, is corrected for spurious background. This was 3.0 cpm in the conventional case - 2.0 cpm for reciprocal measurements. Spurious background for cadmium data was 0.1 conventional and zero for reciprocal.

TABLE VII

BF_3 Data - Shutters Installed

Source Position 1 - Detector Traversed

Tube Posit	N_B	σ_B	N_C	σ_C	N_b	σ_b	N_c	σ_c	N_{th}	σ_{th}
11	908	10	35	2	52	5	2	1	823	11
10	1620	13	58	2	78	6	3	1	1488	14
9	2400	22	92	3	86	7	5	2	2228	23
8	3525	34	130	5	117	8	4	1	3281	36
7	5131	51	192	6	107	7	4	1	4836	52
6	7473	43	260	2	112	1	8	1	7109	43
g	7494	61	254	7	108	7	3	1	7134	62
f	7473	43	260	2	112	1	8	1	7109	43
e	7608	62	281	8	108	8	4	1	7222	63
d	6627	58	225	7	106	8	5	2	6501	58
c	5310	52	190	6	92	7	4	1	5032	52
b	3715	35	127	5	85	7	4	1	3508	36
a	2139	21	77	4	53	5	2	1	2011	22
5	10,959	105	415	9	103	7	5	2	10,446	105
4	17,003	131	704	12	102	7	5	2	16,169	131
3	25,199	159	1316	16	91	7	4	1	23,795	159
2	33,613	183	2438	22	65	6	4	1	31,115	185

Detector Position 1 - Source Traversed

Source Posit	N_B	σ_B	N_C	σ_C	N_b	σ_b	N_c	σ_c	N_{th}	σ_{th}
11	789	9	33	2	36	4	3	1	723	11
10	1346	12	52	2					1261	13
9	2079	24	80	4					1966	24
8	3113	32	118	6					2962	33
7	4544	48	175	9					4336	49
6	6650	58	261	2					6556	58
g	6428	57	249	11					6148	58
f	6650	58	261	2					6356	58
e	6282	56	238	11					6011	57
d	5414	52	212	10					5169	53
c	4108	45	161	9					3914	46
b	2972	31	107	9					2832	32
a	1670	18	59	3					1578	18
5	10,177	101	389	9					9755	101
4	15,420	124	670	26					14,717	127
3	22,895	151	1287	36					21,575	155
2	29,172	170	2421	49					26,718	177

TABLE VII (Continued)

BF_3 Data - Shutters Installed

Source Position 11 - Detector Traversed

Tube Posit	N_B	σ_B	N_C	σ_C	N_b	σ_b	N_c	σ_c	N_{th}	σ_{th}
10	35,981	189	2417	22	78	6	3	1	33,489	190
9	27,852	167	1368	17	86	7	5	2	26,403	168
8	18,313	135	716	12	117	8	4	1	17,483	135
7	11,937	109	432	7	107	7	4	1	11,402	109
6	8058	63	290	5	112	1	8	1	7663	63
g	8006	63	264	5	108	7	3	1	7637	63
f	8058	63	290	5	112	1	8	1	7663	63
e	8116	64	273	5	108	7	4	1	7738	65
d	7204	60	229	5	106	7	5	2	6874	61
c	5650	53	184	4	92	7	4	1	5378	53
b	4020	37	136	4	85	7	4	1	3803	38
a	2479	22	81	3	53	5	2	1	2347	23
5	5453	52	196	4	103	7	5	2	5159	52
4	3816	36	129	4	102	7	5	2	3590	37
3	2508	22	85	3	91	7	4	1	2335	23
2	1613	15	59	2	65	6	4	1	1493	16
1	774	9	28	2	36	4	3	1	713	10

Detector Position 11 - Source Traversed

Source Posit	N_B	σ_B	N_C	σ_C	N_b	σ_b	N_c	σ_c	N_{th}	σ_{th}
10	30,907	175	2403	22	46	2	2	0	28,460	177
9	28,260	168	1348	16					26,868	168
8	18,919	137	731	12					18,144	137
7	12,740	113	431	9					12,265	113
6	8378	37	270	7					8064	38
g	8027	63	271	7					7712	63
f	8378	37	270	7					8064	38
e	7928	63	272	7					7612	63
d	6708	58	230	7					6434	58
c	5374	52	171	9					5159	53
b	3674	35	129	7					3501	36
a	2145	21	77	4					2024	21
5	5688	53	178	9					5466	54
4	3904	36	131	7					3729	37
3	2569	23	84	4					2441	23
2	1620	15	55	3					1521	15
1	937	10	33	2					860	10

TABLE VII (Continued)

BF_3 Data - Shutters Installed

Source in Position 6f - Detector Traversed

Tube Posit	N_B	σ_B	N_C	σ_C	N_b	σ_b	N_c	σ_c	N_{th}	σ_{th}
11	7810	63	282	5	52	5	2	1	7478	63
10	12,197	110	534	7	78	6	3	1	11,588	110
9	23,072	152	932	10	86	7	5	2	22,059	152
8	35,174	187	1693	13	117	8	4	1	33,368	187
7	47,659	218	2933	24	107	8	4	1	44,623	220
5	48,479	220	2959	24	103	7	5	2	45,422	223
4	36,423	191	1722	19	102	7	5	2	34,604	191
3	23,346	152	910	10	91	7	4	1	22,349	152
2	14,453	120	496	7	65	6	4	1	13,896	120
1	6548	81	258	5	36	4	3	1	6257	81

Detector in Position 6f - Source Traversed

Source Posit	N_B	σ_B	N_C	σ_C	N_b	σ_b	N_c	σ_c	N_{th}	σ_{th}
11	8112	64	285	12	112	1	5	0	7720	65
10	14,974	122	507	22					14,360	124
9	24,815	157	908	30					23,800	160
8	37,077	192	1735	42					35,235	197
7	49,947	223	2940	54					46,900	229
5	50,203	224	2897	54					47,199	230
4	36,122	190	1673	41					34,342	194
3	23,024	152	904	30					22,013	154
2	13,971	118	518	23					13,346	121
1	7488	86	258	12					7123	87

Notes:

(1) N_B has been corrected for resolving time.

(2) N_{th} is not corrected for diffusion harmonics or end leakage

(3) All data are rounded to the nearest whole count.

TABLE VIII

Counter Stability Tests

Stability checks were made prior to each complete run, using total bare count as the basis for comparison. In the loaded and unloaded pile, without shutters, stability was checked with the source in the pedestal, the detector at station 6(f); in the loaded pile with shutters installed, stability was checked with the source at 6(f), the detector at 1:

Run	Condition	Total counts per minute (bare)
1	Loaded	2027 ± 18
2	"	2043 ± 14
3	"	2038 ± 14
4	"	2047 ± 15
5	Loaded, shutters installed	6548 ± 81
6	" " "	6629 ± 82
7	" " "	6683 ± 82
8	" " "	6533 ± 81
9	" " "	6650 ± 58
10	" " "	6568 ± 81
11	" " "	6521 ± 81
12	Unloaded	851 ± 9
13	"	874 ± 12
14	"	881 ± 12
15	"	858 ± 12

APPENDIX II

STATISTICAL EVALUATION OF RESULTS

The purpose of this appendix is to discuss in some detail the statistical treatment of data taken during the experiment and to justify the conclusions by means of recognized statistical methods.

A. GENERAL

As previously mentioned, an attempt was made to achieve 1% counting statistics throughout the series of measurements. In certain cases, however, a total count of 10,000 would have required a prohibitive amount of time. In addition to variation in counting statistics, it was necessary to consider the unequal weighting which must be applied to equally valid points on a logarithmic plot. It is desired to find the relationship between standard error σ and weighting factor w such that the squared deviation of the weighted mean is minimized. This is the common problem of the constrained extreme, and it can easily be shown that with

$$\sum_i w_i = n$$

that the squared deviation is minimized in general when

$$w_i = \frac{K}{\sigma_i^2}$$

Now since the standard error of a logarithm is just the fractional standard error of the antilog ($d(\log_e x) = \frac{dx}{x}$) it follows that

$$w_i = \frac{Kx_i^2}{\sigma_i^2}$$

which is identically the inverse of the variance in the logarithm of x.

The standard error, or standard deviation of the mean, has been used throughout this thesis in presenting results.

B. DETERMINATION OF SLOPES

In the least-squares curve fitting procedure, stations 3 through 11 were used in calculating the values of parameters summarized in Appendix I. This was done to allow a direct comparison of numbers obtained by the two methods. Of the four slopes used as bases for computation, however, only the data taken on the loaded pile by conventional methods showed a marked deviation from linear regression. This occurred at stations 1 and 2, where the detector was located within the slowing-down region and near a reflecting interface. The fact that proximity of source to detector did not affect the linearity of the other measurements was verified by least-squares analyses using all points. In these three cases, no significant change in slope resulted.

The equation used to find the regression coefficient

for minimum squared error is

$$m = \frac{\sum w_i z_i y_i - n\bar{z}\bar{y}}{\sum w_i z_i^2 - n\bar{x}^2}$$

where m = slope in linear relation $\log_e(N_{th}) \equiv y = mz + c$

w = weighting factor

$n = \sum w_i$

$\bar{z} = \dfrac{\sum w_i z_i}{n}$

$\bar{y} = \dfrac{\sum w_i y_i}{n}$

The lines shown in figures 6 and 9 were constructed by passing a line of slope m through the intersection of $\bar{x}$ and $\bar{y}$.

C. ERROR

The error in slope was found taking the partial derivative of m with respect to y, using the standard error in the logarithm as the error in y.

The error in regression coefficient found by this method clearly is a function of the counting statistics alone, and the slightly smaller error quoted for the reciprocal method reflects only the increase in counting rate already mentioned. A similar error could be obtained for either method with sufficient time. Much more significant is the residual, or the square of the difference between an observed point and the corresponding point predicted by the least-squares slope.

The weighted mean squared errors are tabulated here for comparison:

$$\frac{\sum w_i(\log_e(N_{th}) \text{ observed} - \log_e(N_{th}) \text{ calculated})_i{}^2}{\sum w_i}$$

	Conventional	Reciprocal
Loaded	4.50×10^{-4}	1.27×10^{-4}
Unloaded	3.27×10^{-4}	1.30×10^{-4}

D. CORRELATION OF DATA

An additional value which relates to precision of measurement is the value of the slope itself. The student's "t" test, while not devised for this exact application, will give a measure of the degree of correlation of the points with a postulated straight line (G-2). By definition,

$$t = \frac{m}{\sqrt{\dfrac{\sum(\delta y)_i^2}{(F-2)\sum(z_i-\bar{z})^2}}}$$

where

$$\delta y = \left[\log_e(N_{th}) \text{ observed} - \log_e(N_{th}) \text{ calculated}\right]$$

and F is the number of degrees of freedom, 9 in this case. Values of "t" obtained from the data are tabulated for comparison.

Degree of Data Correlation (t)

	Conventional	Reciprocal
Loaded	82.4	201.5
Unloaded	141.4	212.0

One should note that the statistical parameters mentioned thus far can be somewhat misleading. The standard error of the slope is a function only of the counting statistics of the points, and will not be altered by poor correlation. The mean squared error and the "t" test, on the other hand, are functions only of correlation and contain no information on the validity of the points. Considerable research has disclosed no parameter which is a function of both of these variables. Therefore the authors propose the following test of correlation:

1. Assume a normal distribution of measurements of the mean exists about a "true" mean value predicted for a point by a least-squares regression coefficient.

2. Define $$\overline{\left(\frac{u}{\sigma}\right)} \equiv \frac{1}{n} \sum \frac{u_i}{\sigma_i}$$

   where u_i is the i^{th} deviation from the predicted value, σ is the standard deviation (or error) determined by total count, and n is the number of points considered.

3. Evans (E-1,p.750), shows the probability of exceeding any given u/σ in a normal distribution. From this, the data correlation may be judged by the probability that $(\overline{u/\sigma})$ will be exceeded in subsequent experiments. For the loaded pile, the dispersion of conventional data shows a probability of being exceeded of only about 15%. More dispersion could be expected from reciprocal data in 37% of further experiments. A value near 50% would be expected of ideal precision.

E. SIGNIFICANCE OF DATA

In justifying the acceptance of reciprocal data from the two lower stations (pile loaded), a normal distribution of experimental results is assumed about a value predicted by

the slope and the intersection of weighted mean values.

	Calculated Value	Observed Value
Sta. 2	9.3080 ± .0085	9.3119
Sta. 1	9.6159 ± .0070	9.6118

Sta. 2: u/σ = .34; Pu* = .73

indicating that 73% of additional measurements of $\log_e(N_{th})$ at station 2 would fall farther from the calculated mean, 27% closer than value observed.

Sta. 1: u/σ = .585; Pu = .56

with similar interpretations.

F. CORRELATION OF RESULTS

Results of the experiment are compared similarly, assuming:

1. The value obtained by the conventional method is the "correct" value
2. Additional measurements by the same method would follow a normal distribution about this "correct" value.

	Conventional	Reciprocal	u/σ	Pu
Material Buckling (cm^{-2})	86 ± 12×10^{-6}	80 ± 11×10^{-6}	.50	.62
Diffusion Length (cm), Unloaded	48.7 ± 1.0	48.1 ± .9	.60	.55

*Pu is defined as the probability that in subsequent measurements of the same value, by means of identical techniques a value in excess of the previously measured value should be obtained. Thus values of Pu are generally expected to fall within the range .90 to .10. The source of most of this material, including notation, is (E-1, Chapters 26-28).

Specifically, these values of Pu indicate that the parameters obtained by the reciprocal method might have been obtained simply by repeating the conventional experiment.

G. SUMMARY

In conclusion, this appendix has shown statistical proof that:

1. The pile parameters measured by the two methods are statistically identical.

2. All points of reference in the pile provide equally valid data except that in the loaded case, data points by conventional measurement within 100 cm of the source do not indicate linear regression.

3 The reciprocal method, with similar statistical accuracy, provides markedly greater experimental precision.

APPENDIX III

DETAILS OF PROCEDURE

This Appendix discusses in detail the experimental methods and calculation procedures used for indium foil measurements.

1. Equipment

Indium foils of 7/8" diameter were punched from .005" stock. Cadmium covers used in measuring the epithermal flux component were 1" in diameter and .020" thick. In order to minimize effects of counter background, beta radiation from the foils was employed as a measure of activation. Counting was done with a proportional flow counter of 2π geometry. The counter is manufactured by Nuclear Chicago and uses a 90% argon, 10% methane gas. This counting method possesses advantages of high efficiency, stable geometry, and excellent time resolution. The counter was connected to an automatic sample changing mechanism which was preset for 1000 or 400 counts, depending upon the expected degree of foil activation. Counter stability was checked frequently by measurement of a standard laboratory beta source. Sufficient counter background measurements were made to establish this factor with essentially no error.

2. Calculation Methods

A printed tape received from the changing and recording mechanism provides the exact time required for a preset number

of counts from each foil. Elapsed time since removal from flux is easily obtained from this, knowing precisely the time required for sample changing. If x' is the time required to obtain a given number of counts with background present and z is the time required for the same number of counts from counter background alone, it can easily be shown that

$$x = \frac{x'z}{z - x'}$$

where x is the time which would be required for this number of counts without background.

Since the neutron flux desired is proportional to saturation foil activity, it is necessary to correct further to allow for the following:

a. Lack of complete saturation activity due to inadequate exposure time.

b. Decay of radioactive isotope during the period following removal from flux and prior to counting.

c. Decay during the counting interval.

The mechanics of these corrections are well known and do not require discussion. For measurements reported in this thesis, the 13 second isotope of indium was allowed to decay before counting and only the 54.2 minute activity was measured.

A form of the decay equation suitable for repetitive calculation is:

$$A_o \lambda_a = \frac{N \lambda_a e^{\lambda_a t_1} e^{\lambda_a x}}{e^{\lambda_a x} - 1}$$

where:

$A_o\lambda_a$ = saturation activity (CPM)

N = preset counts

λ_a = decay constant of 54.2 minute isotope

t_1 = elapsed time from foil removal to start of counting period

x = counting time without background

This expression assumes that exposure time was sufficient to obtain saturation of the isotope being measured. Such was the case for all data included in this report.

After corrected saturation activities were obtained, standard methods were used to reduce these data to relative neutron flux density. A convenient method for this is reported by Wilson, et al (W-3):

$$N_{th} = N_B - N_C - \frac{CR-1}{CR} N_b$$

where cadmium ratio is measured with source in position. The above expression simply assumes that CR does not change in the no-source condition. This supposition is not valid, but its use avoids cadmium covered measurements of the spontaneous fission pile background. These generally are not very satisfying from the statistical point of view. The simplification is justified by the fact that error introduced thereby is very small.

3. Results

Counting rates from indium decay induced by thermal neutrons

varied from 12.5% to 71.7% of the total registered count. Hence statistical variation in the measurements rendered them useless for accurate calculation of "control" parameters. It was not considered worthwhile to correct foil results for slowing down effects, diffusion harmonics, or end leakage. The data, as reported in Table IV, are corrected to relative thermal neutron density. Foils were counted front and back to eliminate shadowing effects. Certain refinements (D-2, M-3, T-1) which might have been applied were not statistically justified. No attempt was made to calculate pile parameters from foil measurements.

APPENDIX IV

SUPPLEMENTARY DISCUSSION

The purpose of this appendix is to discuss qualitatively some aspects of the experiment which do not bear directly upon the objectives of this thesis.

Several features of the experiments conducted by conventional and novel means warrant consideration. There is a separation of lines in figure 9, where thermal neutron density in the unloaded pile measured by the reciprocal method was about 10% greater than that measured by the conventional method. Second, there is a separation of lines in figure 6, showing that in the loaded pile, an increase of about 77% of conventional thermal neutron count rate was observed in the reciprocal experiment. In addition, the conventional method produced a disproportionate increase in counting rate at stations 1 and 2 in the loaded pile which was not observed using the reciprocal method. Third, there is a depression at the center of the conventional horizontal traverses (figures 5 and 8) which was not observed by the reciprocal method. Finally, with both source and detector in the lattice and with cadmium shutters installed, there is a difference in count rate between the two methods, which reverses as the reference point is changed from station 1 to station 11. The authors have found no rigorous explanation of the latter two phenomena. In relation to the deviation of horizontal distributions from the expected cosine, the authors have (1) investigated

the pile for inclusions, slots, irregularities of any description (2) attempted fitting curve with harmonics of the fundamental mode of distribution (3) rechecked raw data. None of these steps have revealed the cause of the deviation.

The first of these conditions is ascribed to differences in net neutron loss from the system. Such an effect would be expected where the system is not isolated from its surroundings. The pedestal rests on a felt base on the concrete foundation; there is no cadmium between the graphite and the foundation of the building. Taking this 10% difference in neutron density as a function of geometry and surroundings, we would expect the factor to apply similarly to the loaded assembly.

As shown by figure 8, the separation of actual count rates is much greater in the loaded assembly than the geometry factor would predict, i.e., the apparent multiplication of neutrons by loading natural uranium in the lattice is much greater in the reciprocal case than in the conventional case(see Appendix V). This is not predicted by one-group theory, which assumes a thermal source. Time limitations have prevented the authors from performing multigroup calculations which might be expected to disclose the reasons for the increased count.

Part of the increase may be attributed to fast fission of U^{238} caused by source neutrons. This could not be expected to exceed 5% according to calculations which conservatively

assume a fast flux equal to thermal flux.

The streaming effect is not considered to produce a significant difference between loaded conventional and reciprocal counts. The pedestal channel extends through the pile, while the detector channels extend only about 3/4 of the horizontal dimension. The Simon-Clifford equation (GR-3, p. 276) gives a ratio of streaming to total flux of only about 5×10^{-5}, indicating a negligible difference in streaming whether the source is in the pedestal or the lattice.

The great difference in cadmium ratio between conventionally and reciprocally measured neutron densities, shown in figure 10, tells us that the average neutron energy is high in the lattice, diminishes near the interface, and is near thermal in the pedestal. This leads to investigation of average neutron temperature in the two regions. Assuming an equal total neutron population with Maxwell-Boltzmann distribution in both cases, and assuming, from unloaded data, a 1.1 geometrical advantage in the reciprocal case, the reciprocal total count is 50% higher than expected. Suppose this is a result of difference in average energy seen by the 1/v boron. If the higher count reflected a kT of .025 ev, the lower value would mean a cross section of about .67 of "thermal" value, hence a kT of about .056 ev. This corresponds to a neutron temperature of about 380°C, which is not unusual for a reproducing region. It is a mistake, however, to attach too much significance to numbers produced by the assumption

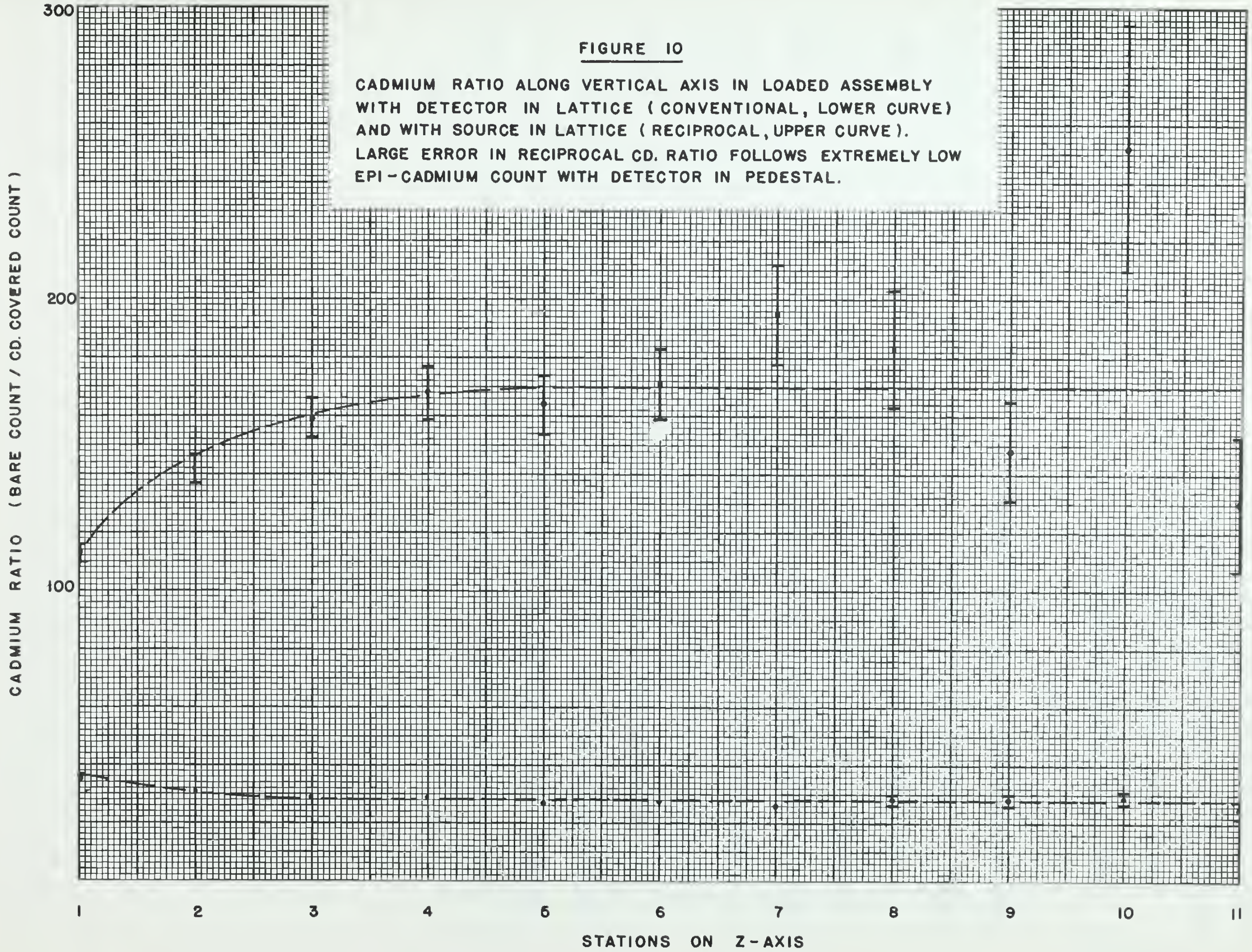


FIGURE 10

CADMIUM RATIO ALONG VERTICAL AXIS IN LOADED ASSEMBLY WITH DETECTOR IN LATTICE (CONVENTIONAL, LOWER CURVE) AND WITH SOURCE IN LATTICE (RECIPROCAL, UPPER CURVE). LARGE ERROR IN RECIPROCAL CD. RATIO FOLLOWS EXTREMELY LOW EPI-CADMIUM COUNT WITH DETECTOR IN PEDESTAL.

of Maxwell-Boltzmann distribution. Many experimenters have attempted to define the very significant deviation of moderated neutrons from this distribution (W-1, p.332, and W-2). There is poor agreement between theory and experiment, and even among experiments.

A difference in flux "hardness" in the vicinity of the detector, then, is supported qualitatively by theory. Physically, we should expect the average neutron energy to be greater in a reproducing region than in moderator. Furthermore, the flux would tend to decrease in average energy near a reflecting interface resulting in the trends observed in figures 6 and 10. The authors are of the opinion that the shift in average neutron energy, evidenced by cadmium ratio, is the primary reason for the differences in counting rates by the two methods. It is cautiously suggested that experiments of this type might be designed to study neutron spectra in lattices.

APPENDIX V

MULTIPLICATION RATIO

The exponential experiment is only one of two basic methods for determination of the critical size of a reactor. The other procedure, which will be briefly considered here, is termed the "critical assembly" method. This method requires the construction of a flexible fuel-moderator assembly which can be gradually increased in size until a self-sustaining reaction is attained. Criticality is predicted by plotting the inverse multiplication ratio versus fuel mass as the loading is successively increased. The fuel mass corresponding to a zero value of reciprocal multiplication ratio (infinite multiplication) indicates the fuel requirement for criticality.

Such a procedure is not feasible with a fixed subcritical assembly of the type used during this thesis. However, it is felt that values of multiplication ratio for the single fixed configuration are of interest and should be reported.

This multiplication can be determined experimentally by measurement of the thermal neutron density within an assembly at a specified distance from the external source. This measurement is made at each location with the pile loaded and subsequently after all fuel has been removed. The ratio of these measurements gives the required multiplication, at a fixed point, for the particular size of the assembly which was investigated.

Such measurements were made by conventional and reciprocal

methods for the M.I.T. subcritical graphite pile. Values were obtained for this ratio of loaded to unloaded thermal neutron density at each of the measurement positions employed throughout the thesis. Figure 11 illustrates the results which were obtained.

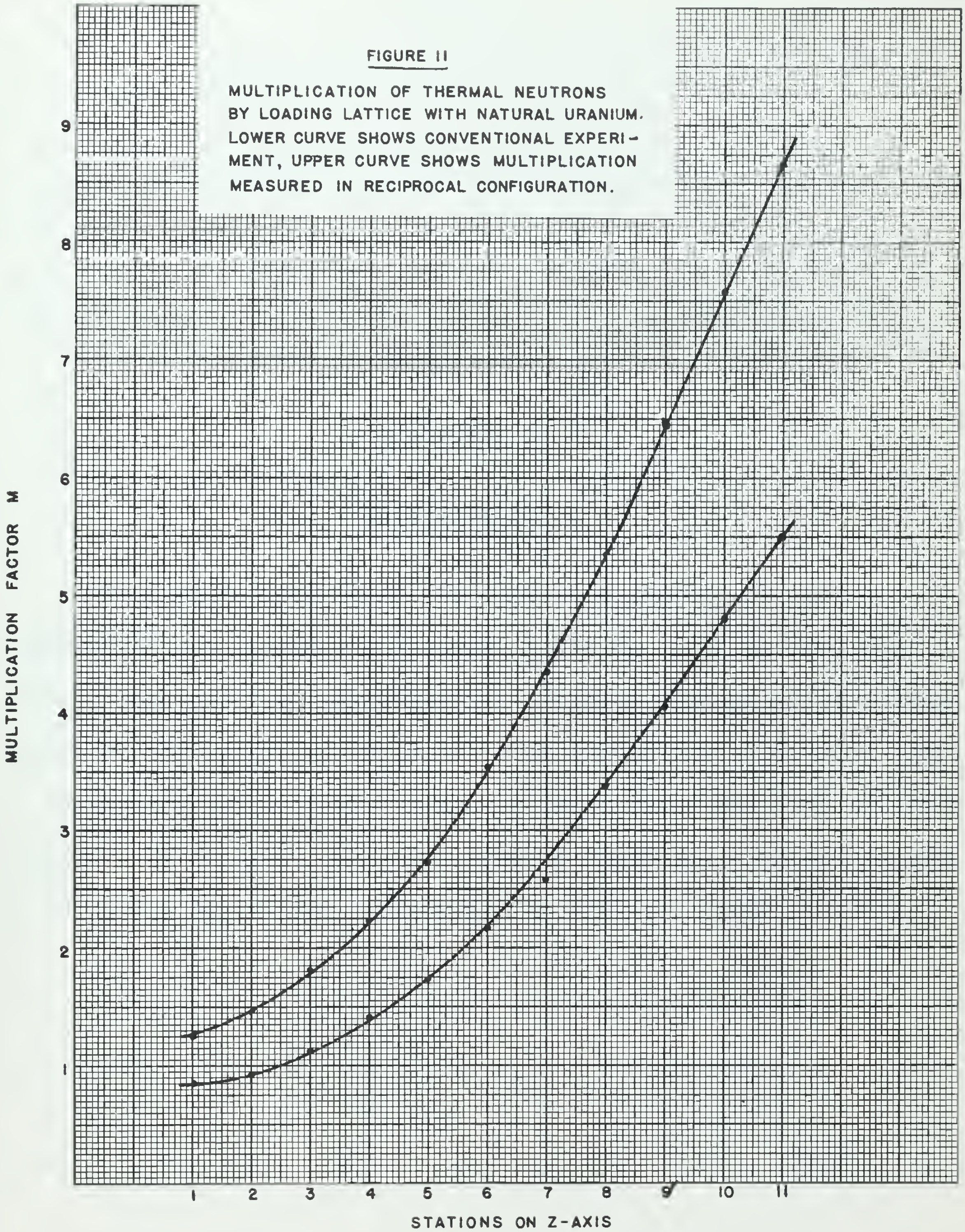


FIGURE II
MULTIPLICATION OF THERMAL NEUTRONS BY LOADING LATTICE WITH NATURAL URANIUM. LOWER CURVE SHOWS CONVENTIONAL EXPERIMENT, UPPER CURVE SHOWS MULTIPLICATION MEASURED IN RECIPROCAL CONFIGURATION.
MULTIPLICATION FACTOR M
1
2
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4
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1
2
3
4
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9
10
11
STATIONS ON Z-AXIS

APPENDIX VI

BIBLIOGRAPHY

The listed references constitute a fairly complete summary of information published prior to 1959 on the subject of exponential experiments. Certain papers are included which relate to reciprocity and the importance function. By no means are all of these works cited within the text of this thesis. It is felt, however, that a thorough compilation may be of value to others contemplating work in these fields.

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